GPS Solut (2011) 15:89–99
DOI 10.1007/s10291-010-0168-6

ORIGINAL ARTICLE

Quality assessment of GPS rapid static positioning with weighted ionospheric parameters in generalized least squares

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Received: 14 August 2009 / Accepted: 27 March 2010 / Published online: 17 April 2010

Abstract Precise GPS positioning requires the processing of carrier-phase observations and fixing integer ambiguities. With increasing distance between receivers, ambiguity fixing becomes more difficult because ionospheric and tropospheric effects do not cancel sufficiently in double differencing. A popular procedure in static positioning is to increase the length of the observing session and/or to apply atmospheric (ionospheric) models and corrections. We investigate the methodology for GPS rapid static positioning that requires just a few minutes of dual-frequency GPS observations for medium-length baselines. Ionospheric corrections are not required, but the ionospheric delays are treated as pseudo-observations having a priori values and respective weights. The tropospheric delays are reduced by using well-established troposphere models, and satellite orbital and clock errors are eliminated by using IGS rapid products. Several numerical tests based on actual GPS data are presented. It is shown that the proposed methodology is suitable for rapid static positioning within 50–70 km from the closest reference network station and that centimeter-level precision in positioning is feasible when using just 1 min of dual-frequency GPS data.

Keywords Ionosphere · Rapid static positioning · GPS · General least squares

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Introduction

Carrier phase-based relative GPS positioning is essential for a wide range of precise applications. This includes a static and kinematic surveying as well as navigation and guidance of a variety of platforms. However, distance-dependent errors, such as satellite orbit and clock biases, atmospheric refractions, reduce the applicable baseline length or increase the observational session length in static positioning. These constraints have stimulated research on new techniques which would eliminate these adverse effects. One of the most advanced techniques recently developed for kinematic positioning is network-based RTK (Network-RTK) including virtual reference stations (VRS) and area correction parameters (ACP/FKP) methods (Vollath et al. 2000; Wanninger 2002; Rizos 2002; Euler et al. 2004; Grejner-Brzezinska et al. 2007; Wielgosz et al. 2008). These methods were also successfully applied in precise static positioning, resulting in a significant decrease of the required observational session length (Kashani et al. 2008; Schwarz et al. 2009). However, these methods require a priori knowledge of the user's approximate position in order to calculate the geometric (tropospheric, orbital) and ionospheric corrections. These corrections are calculated by a reference network processing center based on data from reference stations. In rapid-static or fast-static positioning, when the user position is not required in real time, all the processing may be performed in post-processing. The procedure usually consists of two parts: the first part is the processing of a reference network which allows the derivation of the required corrections; the second one is the processing of the user's data supported by the network-derived corrections.

Previous studies concerned a rapid static positioning with 15 min of dual-frequency carrier phase and

pseudorange GPS data, and their results have led to development of National Geodetic Survey (NGS) online positioning user service-rapid static (OPUS-RS) Internet-based system (Lazio 2007; Kashani et al. 2008; Schwarz 2008). OPUS-RS is developed and implemented on the assumption that at most 10–15 min of dual-frequency GPS phase and code measurements would be available. Another premise is based on the fact that the current national CORS station separations are about 200 km, which means that baselines exceeding 100 km are expected to be processed. These assumptions force the use of the network-derived corrections.

The approach discussed here does not require any external corrections nor reference network data processing at the processing center. It requires, like more traditional solutions such as RTK, just GPS data from one or more reference stations and all the processing may be performed by the user receiver's internal software or software installed in the user's PC. This in turn does not demand bi-directional communication between the user and the reference network control/processing center even if the position in near-real-time is required. The research focuses on precise static positioning methodology that uses short observing sessions of about 1–5 min to obtain a high-accuracy position within the range of several tens of kilometers from the closest reference station. The analyses presented below provide an application of the modified rapid static algorithms that may be applicable for European Position Determination System (EUPOS) and its realization in Poland—ASG-EUPOS (Bosy et al. 2007).

Methodology

In order to support a rapid-static centimeter-level positioning, fast and efficient ambiguity resolution (AR) is required because the receiver occupation time is significantly shorter than that for conventional static GPS surveying. The time required for finding and validating the integer ambiguities (time-to-fix) is a function of baseline length, number of satellites being tracked, and satellite geometry. Aiming toward improvement of rapid static surveying, numerous tests employing different algorithms and software using dual and single-frequency receivers were carried out in the last decade (e.g. Lachapelle et al. 1991; Gianniou 1995; Rizos et al. 1998; Wu and Yiu 1999; Wang 2003). However, these applications are suitable for rather short baselines.

This research aims at ultra rapid-static methodology suitable for medium distances and very short observing sessions. Respective research was originally initiated by Odijk (2000). Earlier studies also include those by Schaffrin and Bock (1988), Teunissen (1997) and de Jong et al. (2000). These authors applied the ionosphere-weighted adjustment models, in which the double-differenced ionospheric delays were treated stochastically instead of deterministically. This allowed taking into account uncertainty of the delays and improved AR.

In the presented study, however, a more general solution based on the general least squares (GLS) is applied (Uotila 1986). It allows applying stochastic constraints on all the parameters such as ionospheric and tropospheric delays, station coordinates, and carrier-phase ambiguities because all the parameters are treated as pseudo observations. The advantage is that the user can conveniently incorporate any a priori information on the adjustable parameters.

Functional model

The functional model uses double-differenced (DD) observables. It is given as

$$\begin{aligned}
&\Phi^{kl}_{1,ij} - \rho^{kl}_{ij} - T^{kl}_{ij} + I^{kl}_{ij} - \lambda_1 N^{kl}_{1,ij} = 0\\
&\Phi^{kl}_{2,ij} - \rho^{kl}_{ij} - T^{kl}_{ij} + (f_1^2/f_2^2) I^{kl}_{ij} - \lambda_2 N^{kl}_{2,ij} = 0\\
&P^{kl}_{1,ij} - \rho^{kl}_{ij} - T^{kl}_{ij} - I^{kl}_{ij} = 0\\
&P^{kl}_{2,ij} - \rho^{kl}_{ij} - T^{kl}_{ij} - (f_1^2/f_2^2) I^{kl}_{ij} = 0
\end{aligned} \tag{1}$$

where i, j are the receiver indexes, k, l the satellite indexes, the DD phase observation on frequency n ($n = 1, 2$), $\Phi^{kl}_{n,ij}$ the DD code observation on frequency n, p^{kl}_{ij} the DD geometric distance (which is a function of user receiver coordinates), T^{kl}_{ij} the DD tropospheric delay, I^{kl}_{ij} the DD ionospheric delay, f_1, f_2 the GPS frequencies of L1 and L2 signals, λ_1, λ_2 the wave lengths of L1 and L2 signals, $N^{kl}_{1,ij}, N^{kl}_{2,ij}$ are the carrier phase ambiguities.

This model requires dual-frequency pseudorange and carrier-phase GPS observations. The unknown parameters are user receiver coordinates, DD ionospheric delays, and DD ambiguities. All the parameters are constrained to some a priori information, which may consist of empirical values (e.g. 5–20 cm for the DD ionospheric delays, depending on the baseline length and local time). The tropospheric delays are obtained from the Modified Hopfield Model (Goad and Goodman 1974) using standard atmosphere parameters and tightly constrained in the adjustment.

The classical three-step solution is used: a float solution with the ambiguities as real numbers in step one, integer ambiguity search in the second step, and fixed solution when real-valued ambiguities are being replaced by the integers in step three. The least-squares ambiguity decorrelation adjustment (LAMBDA) is used to fix the ambiguities to their integer values (Teunissen 1994; de Jonge and Tiberius 1996), and the W-ratio test (Wang et al. 1998) is used for the integer selection validation.

Adjustment model

The adjustment model is based on the GLS algorithm developed by Uotila (1986) in which the conventional least squares adjustment (LSA) model is modified assuming that all parameters comprising the mathematical model are also measured quantities (pseudo-observations). An observation l associated with infinitely large weight becomes constant and the corresponding residual v is zero. On the other hand, an observation with a weight equal to zero becomes an unknown parameter in the classical meaning. Choosing the weights between zero and infinity easily allows setting different constraints on parameters and observations. The mathematical model is expressed by

$$F(L^a) = 0 \tag{2}$$

in which all the parameters are also considered to be observables. All the observations are associated with their corresponding residuals v and their corresponding weights p (Uotila 1986).

Since the parameters are treated as pseudo-observations, each parameter must be associated with its stochastic model, i.e., the variance–covariance matrix (VCV); hence some a priori information on the parameter values and their stochastic properties must be known. This condition is relatively easily fulfilled as described below.

The classical parameters in the presented model are user receiver coordinates, DD carrier phase ambiguities, and DD ionospheric delays. Hence the design matrix A may be constructed of several submatrices related to these parameters: where A_{coord} is the sub design matrix for coordinates (user and reference stations), A_{ambi} the sub design matrix for ambiguities, A_{iono} is the sub design matrix for ionospheric delays.

The approach based on generalization requires one additional sub design matrix A_{obs} for actual observations, which is an identity matrix with size equal to the number of observations. As already mentioned, stochastic properties of all the pseudo-observations must be provided. This allows the respective sub weight matrices to be created: where P_{coord} is the sub weight matrix for coordinates, P_{ambi} the sub weight matrix for ambiguities, P_{iono} the sub weight matrix for ionospheric delays, P_{obs} is the sub weight matrix for the actual observations.

When processing GPS data, an absolute position of the station is usually obtained before processing DD observables. In the presented approach, the DGPS accumulated solution using pseudoranges from all the observing epochs in a session is used as a priori user's position. As a rule, its accuracy is better than 0.50 m. In the numerical tests presented below, however, sigma of 1.00 m for a priori coordinate component was used when constructing a diagonal P_{coord} submatrix. It should be noted that the full VCV matrix resulting from the adjustment of DGPS-derived coordinates may be used as well. A priori sigma for the reference station coordinates was set to 0.005 m. A priori DD ambiguities may be calculated based on the a priori user's position and the actual pseudorange and carrier-phase observations. Their accuracy is usually better than 5–10 L1 cycles and should be reflected in their P matrix.

It is thought that the most important element in the presented methodology is the weight submatrix for DD ionospheric delays, which allows setting different constraints on DD ionosphere. These constraints in form of sigmas must account for baseline length and current ionospheric activity (see "Experiment design"). The a priori values for DD ionospheric delays are set to zero in this approach; therefore, the values of the constraints must correspond to their expected size. However, if external information about the ionosphere is available, e.g., from active reference networks or from IGS IONEX files, the size of the constraints may be reduced, allowing faster AR (Wielgosz et al. 2005; Kashani et al. 2008).

The final, but not less important matrix is the weight submatrix for the actual observations, which is a standard P matrix. It takes into account mathematical correlations between observations due to double differencing or inter-baseline correlations (when adjusting data from several baselines together).

In the presented methodology, the pseudo-observations were divided into two distinct groups: parameters/observations which are accumulated between epochs, denoted with "X" index, and instantaneous parameters/observations that exist only in a particular epoch, denoted with "F" index. The former group includes the user/reference coordinates and the DD ambiguities, and the latter one includes the DD ionospheric delays and the actual DD observables. The instantaneous parameters/observations are eliminated in the sequential adjustment in order to reduce the size of the normal matrix. It should be noted that no correlation between the parameter/observation groups is assumed. Hence, the mathematical model may be modified as (Uotila 1986)

$$F(L_X^a, L_F^a) = 0 \tag{3}$$

and

$$B_F V_F + B_X V_X + W_F = 0. \tag{4}$$

The respective design and weight matrices are constructed as

$$B_X = [-A_{\text{ambi}} - A_{\text{coord}}] \tag{5}$$

where B_X is the design matrix for the accumulated parameters,

$$P_X = \begin{bmatrix} P_{\text{ambi}} & 0 \\ 0 & P_{\text{coord}} \end{bmatrix} \quad (6)$$

where P_X is the weight matrix for the accumulated parameters,

$$B_F = [A_{\text{obs}} - A_{\text{iono}}] \quad (7)$$

where B_F is the design matrix for the instantaneous parameters,

$$P_F = \begin{bmatrix} P_{\text{obs}} & 0 \\ 0 & P_{\text{iono}} \end{bmatrix} \quad (8)$$

where P_F is the weight matrix for the instantaneous parameters.

The objective function will be as follows:

$$\Phi = V_F^T P V_F + V_X^T P V_X - 2K_F^T (B_F V_F + B_X V_X + W_F). \quad (9)$$

The above notation is completed with W_F, vector of misclosures (observed minus computed) and K_F, Lagrange multipliers (Uotila 1986). Note that similar models were also explored in Leick (2004, p. 121).

The essential feature of the GLS model is its flexibility and convenience of implementing different stochastic constraints, weighted parameters, and fixed constraints. The presented methodology a high-quality float solution with accurate float ambiguity estimates when using short data spans. This in turn makes it easy for LAMBDA method to quickly find correct integers.

Experiment design

In order to test the proposed methodology two test networks of different spatial extent were analyzed, called the small and large network. The test networks were constructed by using several GPS stations of the Polish part of EUPOS active geodetic network called ASG-EUPOS (Fig. 1). Stations TARG, WODZ, and KRAW constitute the small network with inter-station distances of 58–87 km. Station KATO which is located inside this small network was chosen as a simulated user receiver with distances to the reference stations of 25, 50, and 67 km, respectively. The large network consists of stations WODZ, LELO, and KRAW with inter-station distances of 105–113 km. Once again, station KATO simulated user receiver with distances to the reference stations of 50, 64, and 67 km, respectively. The small network was designed in such a way that it closely reflects the geometry and size of ASG-EUPOS with average station separation of 70 km (Bosy et al. 2007). The large network was designed to enable analysis of a sparser network with average station separation of over 100 km, which provides a more challenging scenario. This geometry allowed the testing of two cases with the distance between the rover and the closest reference station equal to 25 and 50 km for the small and large reference network, respectively. EUPOS stations provide dual-frequency pseudorange and carrier-phase GPS data with a 1-s sampling rate. A 5-s sampling was used in the experiment. The stations were equipped with Thales/Ashtech microZ receivers and choke-ring antennas with snow covers. A 24-h data set was collected on May 8, 2007. During that day the ionosphere was active but not stormy with maximum K_p index of 4o (Bartels 1957).

In the numerical experiments presented here, the user's position was calculated with using short data spans from the small and large reference networks. In addition, the user's position was derived by two independent solutions based on a single- and a multi-baseline mode. In the multi-baseline mode, the data from the user receiver and the three surrounding reference stations were processed in a single adjustment step. All the processing was performed by GINPOS software developed from the MPGPS package (Wielgosz et al. 2005; Kashani et al. 2008).

Test results

The first analysis deals with the actual DD ionospheric delays. The position results are then presented separately for the small and large networks using 5- and 1-min data sets for the shortest baseline and multi-baseline scenarios.

The ionosphere

Figures 2 and 3 present the DD ionospheric delays over the baselines connecting the user receiver with the reference stations. As it has already been mentioned, the ionosphere was in its typical, moderate state (max $K_p = 4o$).

Figures 2 and 3 show that for the shortest baseline (25 km), the actual DD ionospheric delays usually did not exceed ±5 cm, which is about one fourth of the L1 cycle length. These values of delays usually do not disrupt the AR process dramatically, however if the DD ionospheric delays are included in the mathematical model as parameters, it may improve the speed and reliability of the AR. When studying the longer baselines (50–67 km), it can be observed that the DD ionospheric delays may exceed even half of the L1 cycle length. This may cause adverse effects on AR and, hence, on the position quality and reliability. It is believed that the proposed methodology is able to address these adverse affects. It should be mentioned that during the ionospheric storms DD ionospheric delays can amount to ±2 cycles of L1 over such distances (Wielgosz et al. 2005; Jakowski et al. 2008).

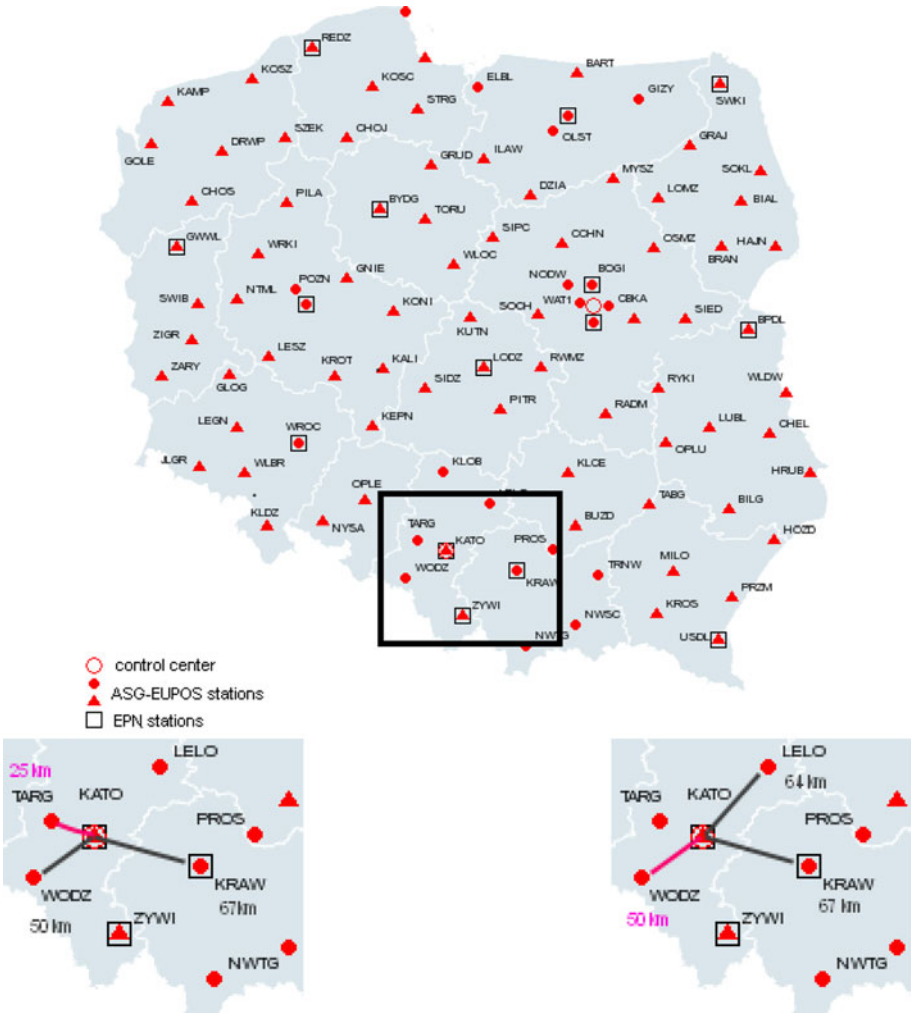

Fig. 1 Small (*left*) and large (*right*) test reference network locations. The *red line* shows the shortest rover-network baseline (map: http://www.asgeupos.pl)

Small network results

In the first positioning test, the user's position was derived using the data from the closest reference station, i.e., KATO-TARG (25 km) baseline. The 24-h data set was divided into 288, 5-min sessions. The sessions were processed independently, yielding hence 288 solutions. In order to test the quality of the solution, several metrics were analyzed, such as

- the position residuals to the reference user's position, their mean, and standard deviation (repeatability),
- ambiguity resolution success ratio, defined as number of sessions where the ambiguities were correctly fixed and validated to the total number of processed sessions,
- ambiguity validation failure ratio, defined as number of sessions where the wrong ambiguities passed the discrimination test to the total number of processed sessions.

The reference position was derived using 24 h of static user's data processed with Bernese 5.0 software (Dach et al. 2007).

Figure 4 presents position residuals obtained for processing the KATO-TARG baseline using 5-min data spans. The majority of horizontal positions fall within the predefined ±2 cm range. Vertical position residuals also usually fall within the predefined ±5 cm limit. The ambiguities could be correctly resolved in 99.6% of the sessions (Table 1). In the case of a single session, few wrong ambiguities passed the discrimination test (0.4%), resulting in slightly worse precision of the position (black dots in Fig. 4). Overall, owing to fact that no external corrections were applied, the results are very promising.

For the next test we added several longer baselines to the solution. Figure 5 presents position residuals obtained from multi-baseline solution (25, 50 and 67 km baselines).

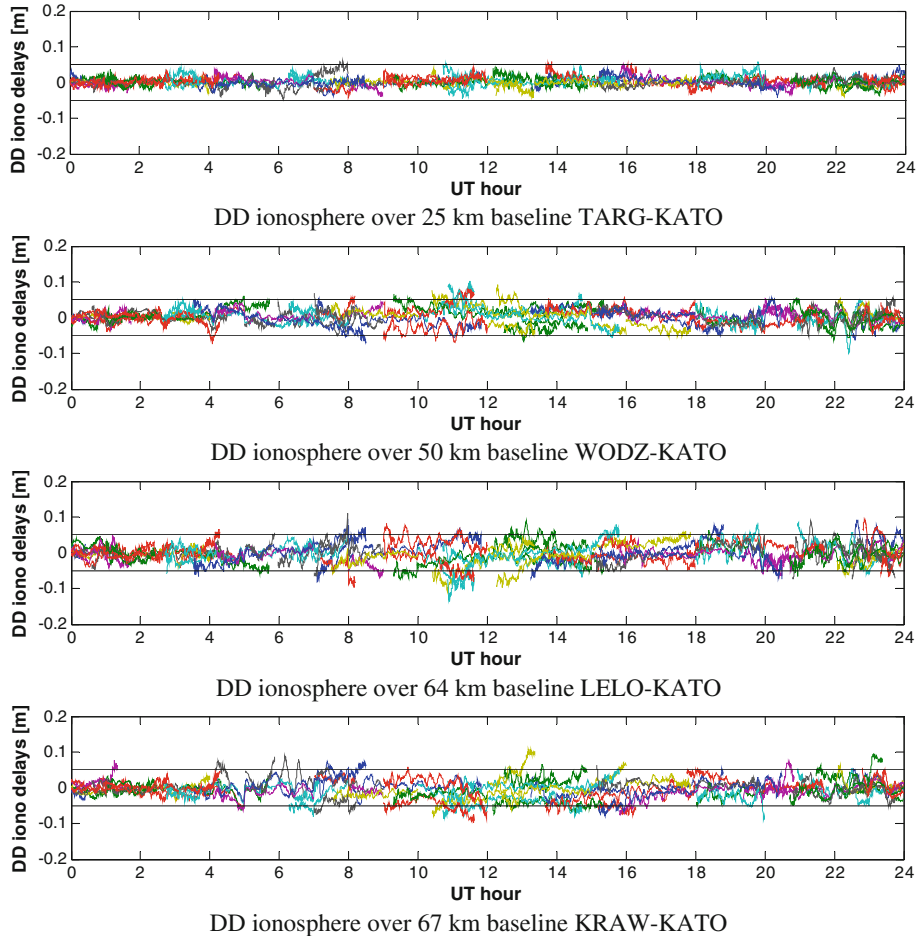

Fig. 2 Actual DD ionospheric delays over the processed baselines observed over 24-h period (*different colors* denote different DD observables)

These results show that the quality of the solution is slightly improved. Table 1 confirms that the repeatability of the 288 solutions is better. But, what is most important, the reliability of the solution reached 100%, i.e., ambiguities in all the processed sessions were solved correctly.

These encouraging results led to the idea of processing even shorter observing sessions. Therefore, the length of the session was reduced to just a single minute (12 epochs with 5-s interval) and the processing scenarios described above were repeated. Figure 6 and Table 1 show that in case of a single-baseline scenario the results are comparable to those obtained from 5 min of data. The position quality was slightly worse and AR was successful again at 99.6%.

Figure 7 presents the results of multi-baseline processing using of just 1 min of data. Analyses of the multi-baseline results lead to the conclusion that the addition of more baselines, even much longer ones, improves the quality of the solution. This is due to the availability of the larger number of the observations, which make the adjustment more reliable. The multi-baseline approach allowed solving the ambiguities in all the processed sessions, providing a high quality position with just 12 epochs of GPS observations.

Large network results

The analysis for the large network, where the user receiver was located 50 km from the closest reference station, is described below. This is a more challenging scenario since the user-reference distance is doubled. Figures 2 and 3 show that the DD ionospheric delays observed over a 50-km baseline may exceed 10 cm. Again, the user's data were processed using 5-min and short 1-min sessions in both single- and multi-baseline modes.

Figure 8 presents the outcome for the KATO-WODZ baseline (50 km) obtained when processing 5-min sessions. The results are comparable to the ones presented in Fig. 4 for 25 km baseline. The majority of the residuals for horizontal position components fall within the ±2 cm range. The AR success rate amounted to 99.3%. The accuracy of the horizontal positions obtained with wrong ambiguities (0.7% of the sessions, black dots in Fig. 8) is still better than 5 cm. This is because the current implementation of

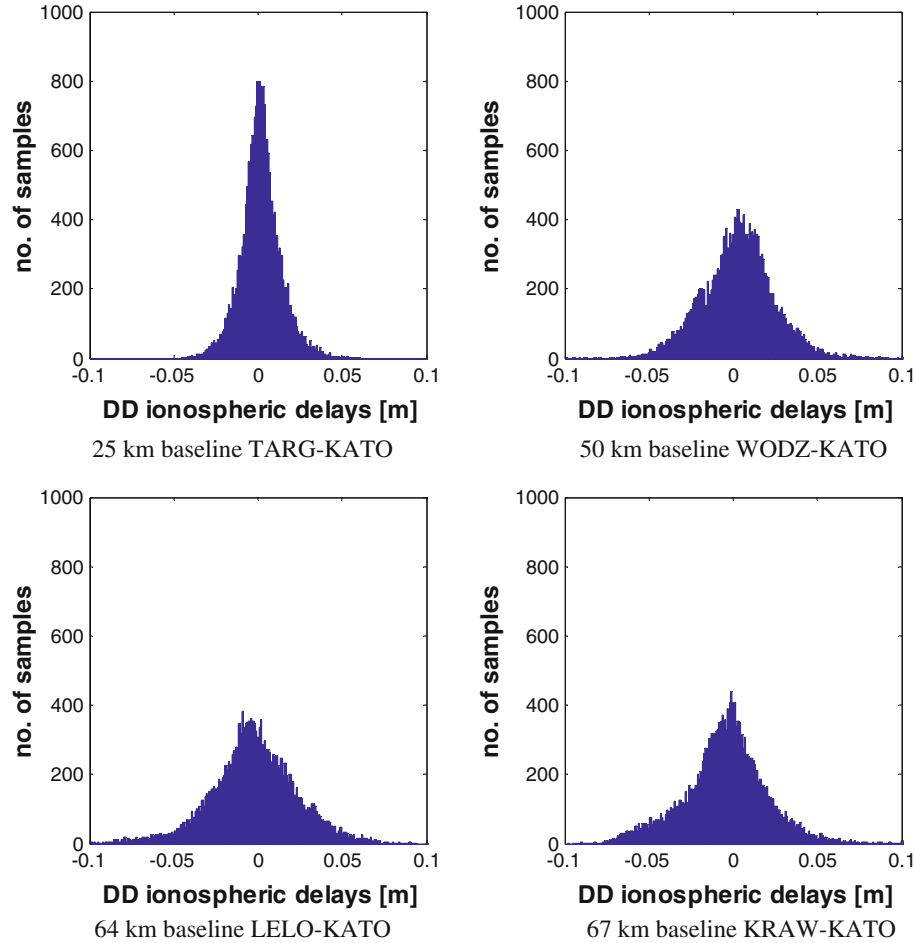

Fig. 3 Histograms presenting DD ionospheric delays over the processed baselines

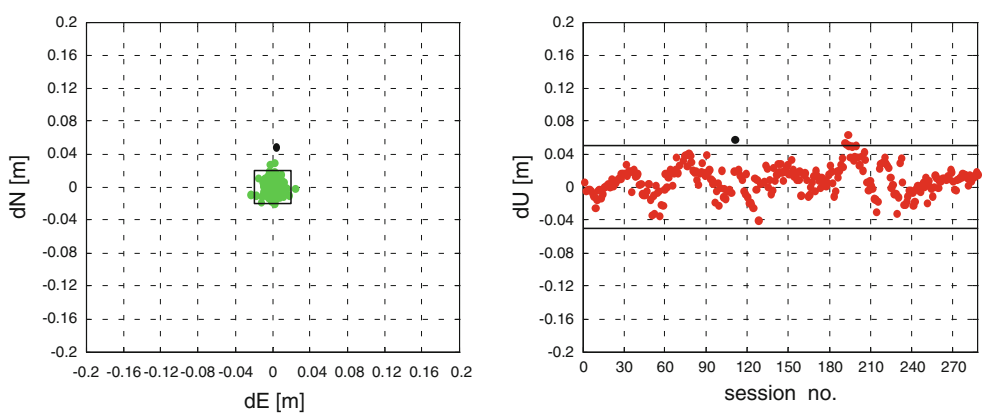

Fig. 4 Position residuals for the KATO-TARG baseline (25 km) and 5-min sessions

Table 1 Mean and standard deviation of the position residuals and AR statistics (small network)

Session length, No. of baselines	Mean dN (mm)	Mean dE (mm)	Mean dU (mm)	Std dN (mm)	Std dE (mm)	Std dU (mm)	AR success rate (%)	AR valid. fail. (%)
5 min, 1 baseline	**1**	**0**	**9**	8	6	17	**99.6**	**0.4**
5 min 3 baselines	**1**	**−1**	**7**	6	4	13	**100.0**	**0.0**
1 min, 1 baseline	**1**	**−1**	**9**	9	6	19	**99.6**	**0.4**
1 min, 3 baselines	**1**	**−1**	**7**	7	5	15	**100.0**	**0.0**

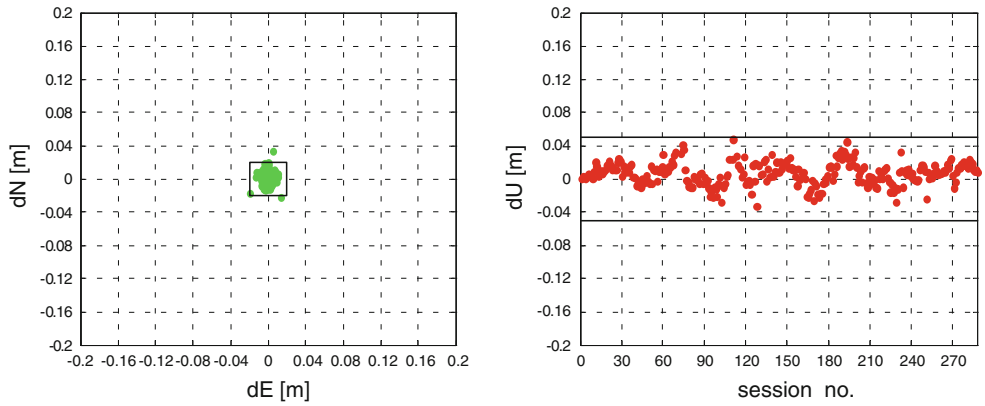

Fig. 5 Position residuals for multi-baseline (25, 50 and 67 km) solution and 5-min sessions

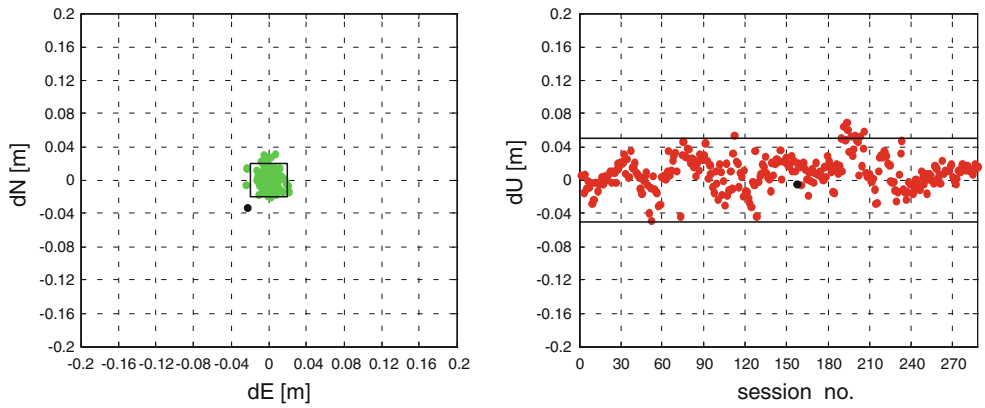

Fig. 6 Position residuals for KATO-TARG baseline (25 km) and 1-min sessions

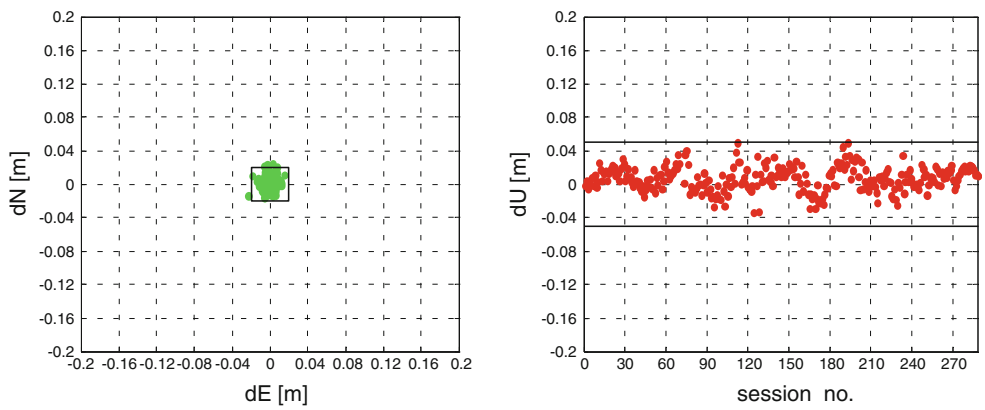

Fig. 7 Position residuals for multi-baseline (25, 50 and 67 km) and 1-min sessions

the W-ratio test has two states: 1, when all the ambiguities in session are fixed and passed the test; and 0, when not every ambiguity passed the test. This means that if a single ambiguity cannot be fixed correctly the test will be marked AR failure. But still the position may be determined relatively well. This is a technical issue that will be addressed in the future.

Figure 9 presents the results of a multi-baseline solution. Compared with the single-baseline, the position quality is better, mostly in terms of repeatability (Table 2). The reliability rate of the solution reached 100%. This confirms that the addition of baselines improves the reliability of the solution.

Figure 10 shows the results of a single-baseline solution when the length of the session was again reduced to a single minute. The obtained position accuracy is still satisfactory. The horizontal residuals are under ±4 cm, and vertical component residuals do not exceed ± 8 cm. The results presented in Fig. 11 and Table 2 confirm that a multi-baseline solution improves the results in terms of successful AR and horizontal and vertical position accuracy.

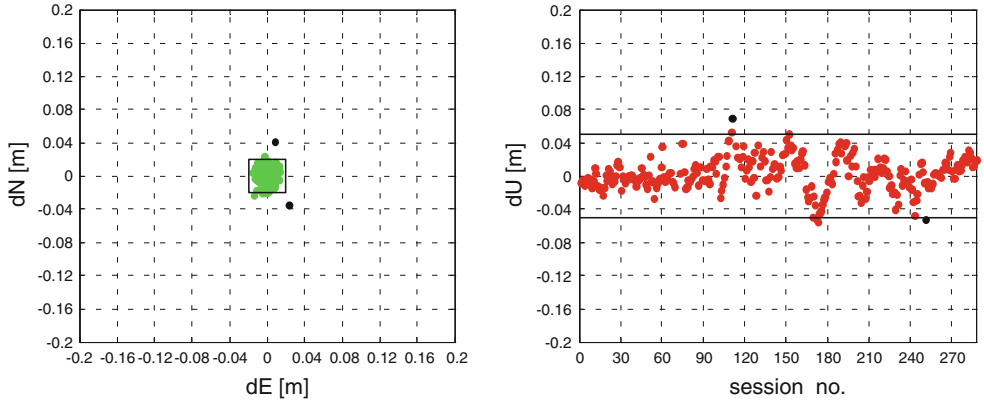

Fig. 8 Position residuals for KATO-WODZ baseline (50 km) and 5-min sessions

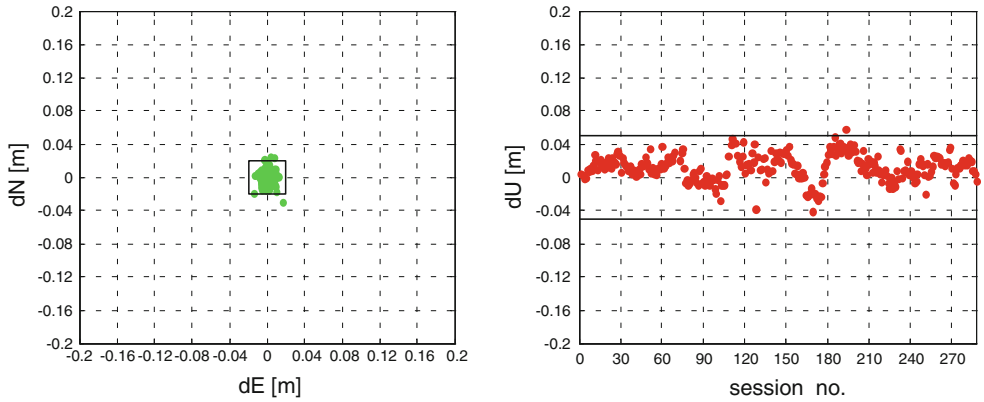

Fig. 9 Position residuals for multi-baseline (50, 64 and 67 km) solution and 5-min sessions

Table 2 Mean and standard deviation of the position residuals and AR statistics (large network)

Session length, No. of baselines	Mean dN (mm)	Mean dE (mm)	Mean dU (mm)	Std dN (mm)	Std dE (mm)	Std dU (mm)	AR success rate (%)	AR valid. fail. (%)
5 min, 1 baseline	**−1**	**−1**	**3**	9	6	20	**99.3**	**0.7**
5 min 3 baselines	**0**	**0**	**12**	6	4	16	**100.0**	**0.0**
1 min, 1 baseline	**−1**	**−1**	**4**	9	6	22	**99.7**	**0.0**
1 min, 3 baselines	**0**	**0**	**13**	7	5	17	**99.7**	**0.0**

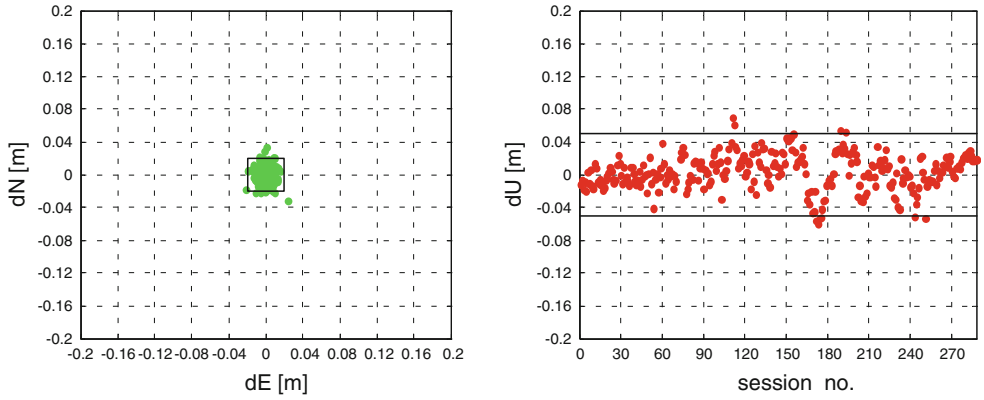

Fig. 10 Position residuals for the KATO-WODZ baseline (50 km) and 1-min sessions

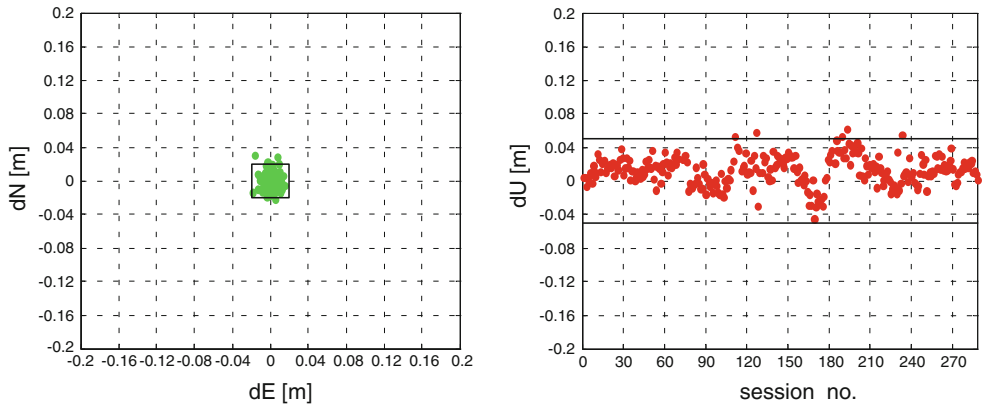

Fig. 11 Position residuals, multi-baseline (50, 64 and 67 km), 1-min sessions

Conclusions

It was demonstrated that GLS adjustment with weighted ionospheric parameters may be successfully applied in precision GPS ultra rapid static positioning. It allows one to obtain accurate and reliable position when using just a 1-min dual-frequency GPS data. This is because the presented methodology provides the high-quality float solution that allows LAMBDA to find the correct integers easily.

These algorithms may be successfully applied in Internet-based on-line positioning services connected with regional reference networks such as ASG-EUPOS. It would significantly shorten the duration of positioning processes, which is essential for users who desire an accurate and reliable 3D position.

References

Bartels J (1957) The technique of scaling indices K and Q of geomagnetic activity. Ann Intern Geophys 4(1):215–226

Bosy J, Graszka W, Leończyk M (2007) ASG-EUPOS—a multifunctional precise satellite positioning system in Poland. Eur J Nav 5(4):30–34

Dach R, Hugentobler U, Fridez P, Meindl M (2007) BERNESE GPS Software Version 5.0. Astronomical Institute, University of Berne, Berne, Switzerland, 612 p

de Jong PJ, Bock Y, Bevis M (2000) Epoch-by-epoch™ positioning and navigation. In: Proceedings of ION GPS-2000, The Institute of Navigation, Alexandria, VA, pp 337–342

de Jonge PJ, Tiberius C (1996) The Lambda method for integer ambiguity estimation: implementation aspects. Delft Geodetic Computing Centre LGR Publication 12, Delft, pp 1–49

Euler H-J, Seeger S, Zelzer O, Takac F, Zebhauser BE (2004) Improvement of positioning performance using standardized network RTK messages. In: Proceedings of ION NTM-2004, January 26–28, San Diego, CA, pp 453–461

Gianniou M (1995) Quality analysis of the GPS observations for rapid-static and on-the-fly applications: problems and solutions. J Satell Position Nav Commun (SPN) 20(4):138–146

Goad CC, Goodman L (1974) A modified Hopfield tropospheric refraction correction model. American Geophysical Union Annual Fall Meeting (abstract), San Francisco, CA, December 12–17, pp 28

Grejner-Brzezinska DA, Kashani I, Wielgosz P, Smith DA, Spencer PSJ, Robertson DS, Mader GL (2007) On efficiency and reliability of ambiguity resolution in network-based RTK GPS. J Surv Eng 133(2):56–65

Jakowski N, Mielich J, Borries C, Cander L, Krankowski A, Nava B, Stankov SM (2008) Large-scale ionospheric gradients over Europe observed in October 2003. J Atmos Sol Terr Phys 70(15):1894–1903

Kashani I, Wielgosz P, Grejner-Brzezinska DA, Mader GL (2008) A new network-based rapid-static module for the NGS online positioning user service—OPUS-RS. Navigation 55(3):255–264

Lachapelle G, Erickson C, Bin SB (1991) Precise rapid surveys using a combination of code and carrier measurements. In: Proceedings of ION GPS-1991, September 11–13, Albuquerque, New Mexico, pp 691–698

Lazio P (2007) Constraining network adjustments to OPUS-RS coordinate observations. J Surv Eng 133(3):106–113. doi: 10.1061/(ASCE)0733-9453(2007)133:3(106)

Leick A (2004) GPS Satellite Surveying, 3rd edn, John Willey & Sons, Inc. Hoboken, xxiv + 435 p

Odijk D (2000) Weighting ionospheric corrections to improve fast gps positioning over medium distances, In: Proceedings of ION GPS-2000, The Institute of Navigation, Alexandria, pp 1113–1123

Rizos C (2002) Network RTK research and implementation: a geodetic perspective. J GPS 1(2):144–150

Rizos C, Han S, Han X (1998) Performance analysis of a single-frequency, low-cost GPS surveying system. In: Proceedings of ION GPS-1998, September 15–18, Nashville, Tennessee, pp 427–436

Schaffrin B, Bock Y (1988) A unified scheme for processing GPS dual-band phase observations. J Geod 62(2):142–160

Schwarz CR (2008) Heuristic weighting and data conditioning in the National Geodetic Survey rapid static GPS program. J Surv Eng 134(3):76–82. doi:10.1061/(ASCE)0733-9453(2008)134:3(76)

Schwarz CR, Snay RA, Soler T (2009) Accuracy assessment of the National Geodetic Survey's OPUS-RS utility. GPS Solut 13(2):119–132

Teunissen PJG (1994) A new method for fast carrier phase ambiguity estimation. In: Proceedings of IEEE PLANS, Las Vegas, NV, April 11–15, pp 562–573

Teunissen PJG (1997) The geometry-free GPS ambiguity search space with a weighted ionosphere. J Geod 71(6):370–383

Uotila UA (1986) Notes on adjustment computation, Part I. Department of Geodetic Science and Surveying. The Ohio State University, Columbus, 168 p

Vollath U, Buecherl A, Landau H, Pagels C, Wagner B (2000) Multi-base RTK positioning using virtual reference stations. In:

Proceedings of ION GPS-2000, 19–22 September, Salt Lake City, UT, pp 123–131

Wang Z (2003) A new approach to ill-conditioned problems in rapid positioning using single frequency GPS receivers. In: Proceedings of ION GPS/GNSS-2003, September 9–12, Portland, Oregon, 2003, pp 1229–1239

Wang J, Stewart M, Tsakiri M (1998) A discrimination test procedure for ambiguity resolution on-the-fly. J Geod 72(11):644–653. doi: 10.1007/s001900050204

Wanninger L (2002) Virtual reference stations for centimeter-level kinematic positioning. In Proceedings of ION GPS-2002, September 24–27, Portland, Oregon, pp 1400–1407

Wielgosz P, Kashani I, Grejner-Brzezinska DA (2005) Analysis of long-range network RTK during severe ionospheric storm. J Geod 79(9):524–531. doi:10.1007/s00190-005-0003-y

Wielgosz P, Krankowski A, Sieradzki R, Grejner-Brzezinska DA (2008) Application of predictive regional ionosphere model to medium range RTK positioning. Acta Geophys 56(4):1147–1161. doi:10.2478/s11600-008-0059-1

Wu J, Yiu FG (1999) Multiple GPS base stations for high-reliability positioning. In: Proceedings of ION 55th AM, June 28–30, Cambridge, MA, pp 333–338

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