M4_on-line. Laminar and turbulent flow. Determination of the coefficient of viscosity by the Stokes method

Topics:

- Fundamentals of mechanics – Newton's laws [1] – Chaps. 5-3, 5-6 and 5-8
- Intermolecular interactions.
- Ideal and real fluids [1] – Chap 14
- The equation of stream continuity and Bernoulli's equation [1] – Chaps. 14-9 and 14-10
- Viscosity
- Laminar and turbulent flow.
- The coefficient of viscosity.
- Poiseuille's equation.
- The Reynolds number and the limit Reynolds number.
- Blood flow. Viscosity of blood.

Description of experiment:

A body moving in fluid is subject to a viscosity force, which decelerate its movement. If e.g. a ball moves in liquid, then a layer of liquid being in direct contact with the ball has the same speed as the ball and pulls neighboring layers of liquid in the same direction. The speed of the layer decreases with the ball – layer distance, therefore layers of liquid move with respect to each other. The viscosity force acts between layers of liquid, therefore the moving ball is subject to the force of viscosity resistance. The relation between the force of viscosity resistance and the velocity of the ball, its radius and liquid properties was discovered by Stokes. It is defined by the formula:

$$F = 6\pi\eta r\upsilon \qquad (1)$$

where:

F – the force of viscosity resistance

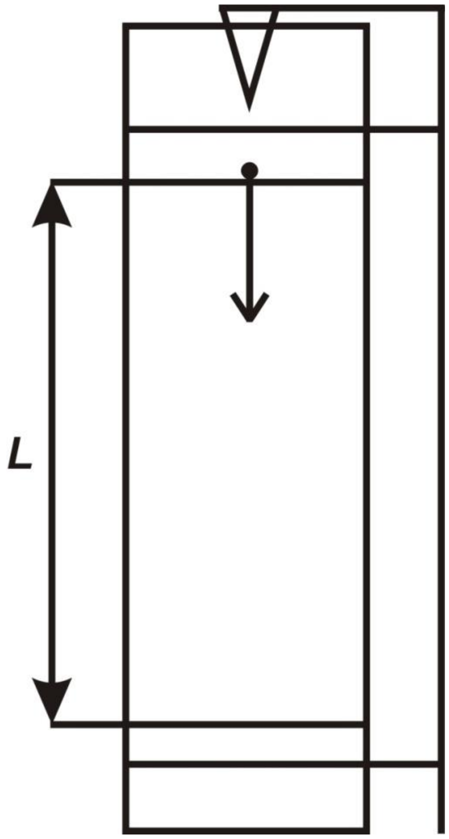
η – the coefficient of dynamic viscosity of fluid

r – the radius of the ball

υ – the velocity of the ball

This formula is correct for laminar flow i.e. when movement of the ball does not generate vortexes and the subsequent layers of liquid move parallel to each other.

The viscosity measurement equipment is made of vertically positioned glass pipe filled with investigated fluid (glycerol). The ball with known density and small radius (compared to radius of the pipe) is dropped into the pipe.



The ball of volume q_k, moving in the fluid, is subject to three forces:

1. The gravity force, acting vertically down:

$$P = mg$$

2. The force of viscosity resistance, acting vertically up:

$$F = 6\pi\eta r\upsilon$$

3. The buoyant force, acting vertically up:

$$W = \varrho_c q_k g$$

Fig. 1.

At the beginning the movement of the ball is accelerated. It is a consequence of the fact that the force P is larger than the sum of forces F+W. The force F increases with increasing velocity of the ball until it reaches the limit value for which its sum with W cancels out the force P. Since then the ball moves with constant velocity and the following equality is fulfilled:

$$F + W = P \qquad (2)$$

The gravity force

$$P = mg = \varrho_k q_k g = \varrho_k \frac{4}{3}\pi r^3 g \qquad (3)$$

where:

ρ_k – the density of the ball

q_k – the volume of the ball

r – the radius of the ball

g – the gravitational acceleration

The buoyant force:

$$w = \varrho_c q_k g = \varrho_c \frac{4}{3}\pi r^3 g \qquad (4)$$

where:

ρ_c – the density of liquid

Substitution of forces (1), (3) and (4) into the equation (2) leads to:

$$6\pi r\eta\upsilon + \varrho_c \frac{4}{3}\pi r^3 g = \varrho_k \frac{4}{3}\pi r^3 g$$

$$6\pi r\eta\upsilon = \frac{4}{3}\pi r^3 g(\varrho_k - \varrho_c)$$

$$\eta = \frac{4\pi r^3 g(\varrho_k - \varrho_c)}{3 \cdot 6\pi r\upsilon}$$

$$\eta = \frac{2r^2 g(\varrho_k - \varrho_c)}{9\upsilon}$$

because:

$$\upsilon = \frac{L}{t}$$

where:

L – the distance passed by the ball

t – time of movement

therefore:

$$\eta = \frac{2r^2 g(\varrho_k - \varrho_c)t}{9L} \qquad (5)$$

Instruction:

Attention:

L – the distance passed by the ball = h - in the Animations,

ρ_c – the density of liquid = ρ_{gl} – the density of water solution of glycerine (in the Animations).

1. There are several types of balls with varying densities for your disposal (each file contains the Animation for one type of ball). Ask the supervisor for the amount and type of balls, which should be dropped into the pipe.
2. Read the interval (distance) of length h (=L in formulas) on the pipe. This is the movement path of the balls.
3. Measure diameter of ball with a Vernier calliper. Determine the radii of the ball (r) and write it down into the table.
4. Drop the ball into the fluid-filled pipe and measure travel time of the ball on the distance h (=L).
5. Use the equation (5) to calculate the coefficient of viscosity of tested fluid.
6. Repeat measurements for subsequent balls.
7. Calculate mean value of the coefficient of viscosity.
8. Enter the results into the table.
9. Compare your results with “true” table value of the coefficient of viscosity for glycerol.

Lp	***r***	***t***	***L (=h)***	***υ***	***η***	η_{av}	***T***
	(m)	(s)	(m)	(m/s)	(Pa·s)	(Pa·s)	(K)

[1] Walker J., Halliday and Resnick, *Principles of physics: international student version*, 9 th ed., extended, Hoboken : John Wiley & Sons, Inc., 2011. , ISBN 978-0-470-56158-4

[…] or other book on physics

M12. Laminar and turbulent flow. Measurement of coefficient of viscosity with Ostwald's viscometer.

Topics:

- Intermolecular interactions
- Ideal and real fluids. Viscosity.
- The equation of stream continuity and Bernoulli's equation.
- Laminar and turbulent flow.
- The coefficient of viscosity.
- Poiseuille's equation.
- The Reynolds number and the limit Reynolds number.
- Blood flow. Viscosity of blood.

Description of experiment:

The Ostwald's viscometer is a capillary viscometer.

The flow of the fluid through capillary pipes is described by Poiseuille's law.

$$\omega = \frac{\pi}{8\eta_c}\frac{p_1 - p_2}{L}r^4 \qquad (1)$$

where:

ω - intensity of flow (volume of liquid flowing out of the capillary during 1 s),

ηc – the coefficient of absolute dynamic viscosity of fluid,

r – the radius of capillary,

L – the length of capillary,

$p_1 - p_2$ – the difference of pressures at both ends of the capillary.

In practice it appears that it is easier to use Poiseuille's formula to determine relative viscosity η, instead the absolute one. Therefore we measure the time of flow of the same amount of tested fluid and distilled water through the same capillary. Then we can write:

$$W = \omega_c t_c = \frac{\pi r^4 \Delta p_c}{8L\eta_c} t_c \qquad (2)$$

$$W = \omega_w t_w = \frac{\pi r^4 \Delta p_w}{8L\eta_w} t_w \qquad (3)$$

where:

t_c – time of outflow of tested fluid of volume W,

t_w – time of outflow of tested fluid of volume W,

Δp_c – the difference of pressures measured at both ends of tested fluid-filled capillary,

Δp_w – the difference of pressures measured at both ends of distilled water-filled capillary

Dividing the equation (2) by equation (3) leads to:

$$1 = \frac{\Delta p_c t_c \eta_w}{\Delta p_w t_w \eta_c} \qquad (4)$$

therefore:

$$\eta = \frac{\eta_c}{\eta_w} = \frac{\Delta p_c t_c}{\Delta p_w t_w} \qquad (5)$$

The flow of fluid through capillary is caused by the gravity forces, therefore the difference of hydrodynamic pressures measured at both ends of capillary is proportional to the density of fluid, therefore:

$$\frac{\Delta p_c}{\Delta p_w} = \frac{\rho_c}{\rho_w} \qquad (6)$$

Substitution of equation (6) into the formula (5) leads to the formula for relative coefficient of dynamic viscosity:

$$\eta = \frac{\rho_c t_c}{\rho_w t_w} \qquad (7)$$

where:

ρ_c – the density of tested fluid,

ρ_w – the density of distilled water.

Instruction

1. The levels *m* and *n* shown in the Fig. 1 should be marked by student (they are not marked on the viscometer). Because the levels determine the volume *W* in equations (3,4) they should remain unchanged for subsequent measurements.
2. Fill the viscometer with distilled water (up to the middle of container Z_2)

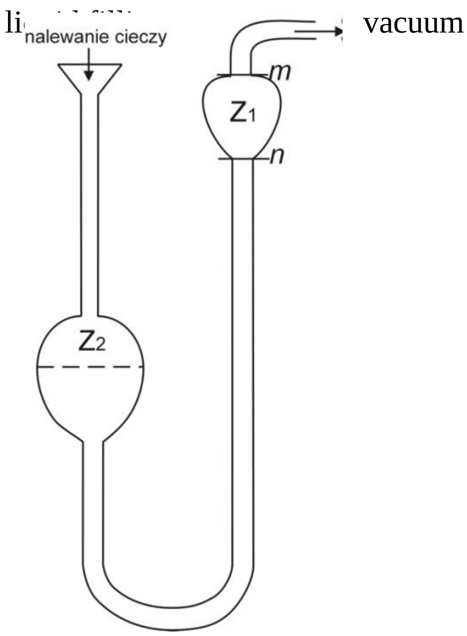


Fig. 1. Ostwald’s viscometer

3. Connect the water pump and use it to pump the water to the upper container Z_1. The whole container should be filled with water (above the level *m*). Be careful, do not pump the water out of viscometer!
4. Switch off the water pump and measure time t_w of the water outflow, during which the water level drops from *m* to *n*. Repeat measurement three times and calculate mean time t_{wav}.. Do NOT add water to the container Z_2 for the subsequent measurements. The difference of water levels in the upper and lower container should remain unchanged.
5. Pour the water out of viscometer.
6. Fill the viscometer with the tested fluid and repeat points 3 – 5.
7. Calculate the relative viscosity from the equation (7) using mean times t_{wav}, t_{cav}. Use the table value of the density of distilled water ρ_w, at the temperature of measurement. The density of the fluid ρ_c is given.
8. Read the table values of absolute viscosity of distilled water, at the temperature of measurement, and calculate absolute viscosity of investigated fluid:

$$\eta_c = \eta \cdot \eta_w$$

9. Enter the results into the table.

No	t_w	t_{wav}	t_c	t_{cav}	ρ_w	ρ_c	η	η_c	T
	(s)	(s)	(s)	(s)	(kg/m^3)	(kg/m^3)		(Pa·s)	(K)