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ACCURACY EVALUATION OF THE SUCCESSIVE CAMPAIGNS OF THE PRECISE LEVELLING IN POLAND

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K e y w o r d s: levelling networks, random errors, systematic errors.

A b s t r a c t

In the paper the evaluation of accuracy of the four successive campaigns of the precise levelling in Poland is presented. Estimation of accuracy was conducted by the: traditional, Lallemand and Vignal formulas. From the conducted evaluations results, that the successive campaigns are characterize by the more and more small random errors. It can be interpreted that better instruments and measuring methods were used in the successive campaigns. However the systematic errors remain almost the same what can be interpreted that the influence of such factors like the topography, climate (refraction) and vertical movements is always the same on the area of Poland. Considerable divergences in the evaluation of systematic errors from Lallemand's and Vignal's formulas suggest that the model of systematic errors adopted by Vignal is probably not realistic and it requires further investigations.

OCENA DOKŁADNOŚCI KOLEJNYCH KAMPANII NIWELACJI PRECYZYJNEJ W POLSCE

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S ł o w a k l u c z o w e: wyrównywanie sieci niwelacyjnych, błędy przypadkowe, błędy systemowe.

A b s t r a k t

W pracy przedstawiono ocenę dokładności czterech kolejnych kampanii niwelacji precyzyjnej w Polsce. Dokładność oszacowano wzorami: tradycyjnymi, Lallemanda i Vignala. Z oszacowań wynika, że kolejne kampanie są obarczone coraz mniejszym błędem przypadkowym, co można interpretować coraz lepszą dokładnością instrumentów i metod pomiarowych, podczas gdy błąd systematyczny pozostaje prawie taki sam, co można interpretować wpływem stałych na danym obszarze czynników, jak topografia, klimat (refrakcja) i ruchy pionowe skorupy ziemskiej. Znaczne rozbieżności w ocenie błędów systematycznych wzorami Lallemanda i Vignala sugerują, że model błędów systematycznych przyjęty przez Vignala jest mało realistyczny i zagadnienie to wymaga dalszych badań.

Introduction

Until now the four campaigns of the precise levelling were realized in Poland. The first campaign in years 1926–1937, the second in years 1953–1955, the third in years 1974–1982 and the last (fourth) in years 1999–2003. The evaluation of the accuracy of these campaigns was conducted using mostly the traditional formulas. The results of these estimates are spread out in many works. For example the accuracy of the first campaign, evaluated by the traditional formulas only, is given in (*Katalog...* 1939). The evaluation of the accuracy of the second campaign by the Vignal formulas is given in (WYRZYKOWSKI 1969) and the evaluation of the accuracy of the second and third campaign in the traditional way and by Lallemand's formulas is given in (WYRZYKOWSKI 1988). The evaluations of the accuracy of the campaign IV by the use of traditional formulas is given e.g. in the paper (GAJDEROWICZ 2005).

The aim of the present work is collection of scattered information concerning networks accuracy and calculation of lacking accuracy evaluation of every campaign if only it is possible. In the first step of the present work successive four campaigns of the precise levelling in Poland are characterized. Then it is given in a large shortcut the basic information relating to the traditional formulas as well as Lallemand's and Vignal's formulas. Next estimation of the accuracy of the campaign I was conducted by the Lallemand's and Vignal's formulas and then estimation of the campaign IV by the traditional, Lallemand's and Vignal's formulas. The results of these calculations, were next completed by estimates from the literature, then were taken down in a suitable tables and introduced on suitable drawings.

Description of the successive campaigns of precise levelling in Poland

The first precise levelling campaign began in 1926 and was finished in 1937. The network consists of 5 907 sections, 121 lines and 36 loops (Fig. 1). Total length of the levelling lines is 10 046 km.

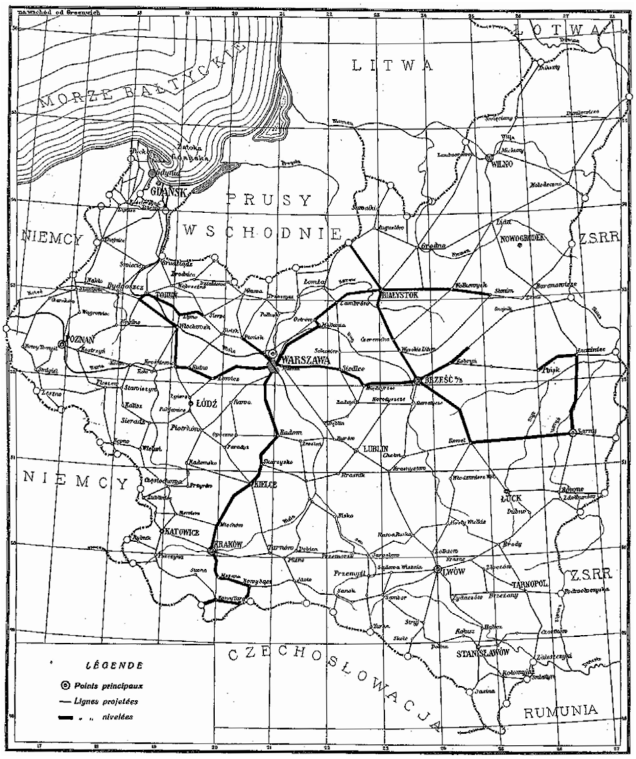

Fig. 1. The levelling network of the first campaign

The levelling lines were measured by the precise optical levels Zeiss III with parallel plate. The observed height differences were corrected due to rod scale and normal orthometric corrections. The adjustment of the levelling network was carried out by the condition method with fixed height of the benchmark in Toruń referred to Amsterdam tide gauge. After adjustment the standard deviation of height difference was $\pm$ 1.04 mm$\sqrt{\text{km}}$ (*Katalog*... 1939).

The second levelling campaign was carried out in two stages. The first measurements were done in 1947–1950, and the second measurements in 1953–1955. The second version of network comprises of 4 500 sections, 60 levelling lines and 12 loops. Total length of levelling lines is 5 778 km. The levelling lines were measured by precise levels Aerogeopribor NA-1 and Wild N III with parallel plates. The observed heights difference was corrected due to rod scale and normal corrections. Gravity necessary to calculate normal Molodensky corrections was referred to Potsdam Gravity System. The first adjustment of the levelling network was carried out by the parametric methods with fixed height of one benchmark in Toruń. The second and final adjustment was carried by parametric method assuming several bench marks as fixed. The results of this adjustments gives the standard deviation of height difference equal ± 0.78 mm$\sqrt{\text{km}}$ (WYRZYKOWSKI 1988). The final results of the adjusted heights of benchmarks are collected in (*Katalog...* 1960) which is described e.g. in (ŁYSZKOWICZ et. al. 2003).

The third levelling campaign was measured in 1974–1982. The network consists of 15 827 sections, 371 lines and 135 loops. The total length of levelling lines is 17 015 km. The levelling lines were measured by automatic levels: Opton Ni1 and Zeiss Ni002. The following corrections were implemented to the raw data: rod scale corrections, rod temperature corrections, tidal corrections, normal Molodensky corrections. The final adjustment of the entire network was carried out in a few versions. In 1985 the accepted solution was obtained as a least square approach with stations constrains. Heights of 23 bench marks with their estimated accuracy (from new UPLN solution) was incorporated to the adjustment. After adjustment the standard deviation of height difference was ± 0.844 mm$\sqrt{\text{km}}$ and standard deviation of adjusted heights changes between ± 6.5 mm and ± 11 mm (WYRZYKOWSKI 1988). The final results of the adjusted heights of benchmarks are collected in (*Katalog...* 1982) which is described e.g. in (ŁYSZKOWICZ et. al. 2003).

The fourth precise levelling campaign started in 1999 and was finished in 2003 (Fig. 2). The network consists of 16 150 sections with average length 1.1 km, 382 lines with average length about 46 km, 135 loops, and 245 nodal points. Total length of levelling lines is 17 516 km. The levelling lines were measured with Zeiss Ni002 (66% measurements), Zeiss DiNi 11 (31% measurements), Topcon NJ (3% measurements) e.g. (PACZUS 2001). As in the case of the third campaign the rod scale corrections, rod temperature corrections, tidal corrections and normal Molodensky corrections were introduce to the raw height differences.

The first, the simplest assessment of a successful network adjustment of the fourth campaign is described in (ŁYSZKOWICZ, JACKIEWICZ 2005). The adjustment of the network was done as the minimally constrained adjustment and the standard deviation of height differences equal ± 0.88 mm$\sqrt{\text{km}}$ was obtained. Identical evaluation of the accuracy of the campaign IV was obtained in the network adjustment carried out in the study (GAJDEROWICZ 2005).

Discrepancies between forward and backward levelling of a section, line and the loop misclosures

In the present study we assume height differences δH of a section or height differences ΔH of a line from the forward and backward levelling as a “observations”.

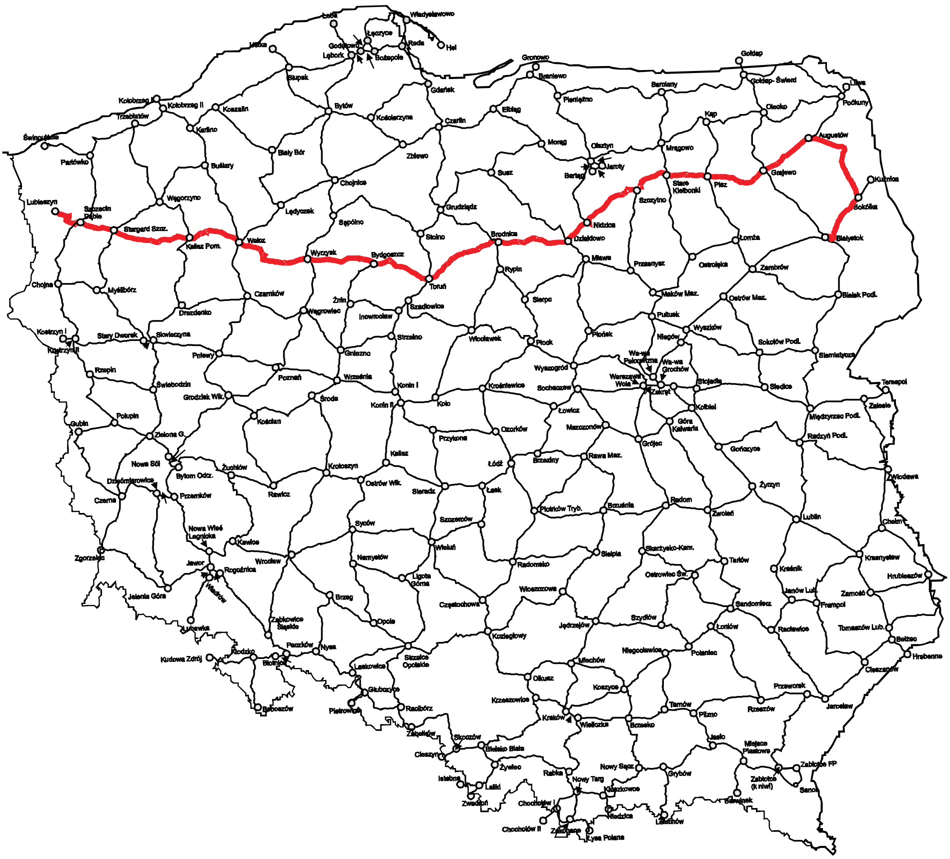

Fig. 2. The levelling network of the fourth campaign

The difference between forward and backward levelling of a section is define by the formula:

$$\rho = \delta H^g - \delta H^p \tag{1}$$

where δH^g is a height difference from the forward measurement and δH^p is a height difference from the backward measurement.

The discrepancy λ of a forward and backward levelling of a line is defined by the formula

$$\lambda = \Delta H^g - \Delta H^p = \sum_{i=1}^{n} \rho_i \tag{2}$$

where ΔH^g is height difference of a line from the forward levelling and ΔH^p is height difference of a line from the forward and backward levelling and n the number of section of a line.

The loop misclosures φ is compute from the formula

$$\varphi = \sum \Delta H_i^{śr} \quad (3)$$

where $\Delta H_i^{śr}$ is a mean height differences of the i line included in the loop (WARCHAŁOWSKI 1954, p. 374) and n is the number of the lines which form the loop.

In a case of the first campaign the discrepancies λ and φ are available only from (*Katalog*... 1939), while for the second campaign we did not have access to any discrepancies. In the case of the third campaign the discrepancies φ are available in (WYRZYKOWSKI 1988) and in the last case all discrepancies are available for the present study. Statistical characteristic of the discrepancies ρ, λ, φ for the successive levelling campaigns are given in Table 1.

Table 1
Statistical characteristic of the discrepancies ρ, λ, φ (in mm) of the successive levelling campaigns in Poland

	ρ	λ	φ
Campaign I			
Number	–	121	36
Mean	–	–2.18	–0.32
Std dev	–	±14.32	±23.56
Min	–	–56.00	–43.80
Max	–	34.50	43.20
Relevant data from campaign II are not available			
Campaign III			
Number	–	–	136
Mean	–	–	–0.39
Std dev	–	–	±14.79
Min	–	–	–36.49
Max	–	–	30.45
Campaign IV			
Number	16 132	382	133
Mean	0.07	2.72	0.27
Std dev	±0.78	±6.88	±12.54
Min	–23.83	–20.41	–31.49
Max	17.72	20.83	28.83

From the Table 1 results that the most data of the section, line and loop misclosures are available for the first and third campaign, and there is no data for the second campaign while all data are available for the fourth campaign.

From the comparison of the loop misclosures of the levelling campaigns results that the mean value of the discrepancies φ in all campaigns are comparable (the same systematic factors), however standard deviation of the discrepancies getting smaller – even twice – what means that the instruments and methods of the measurement are more and more precise.

Accuracy evaluation of the successive campaigns of the precise levelling

Traditional formulas

During the last one hundred and fifty years several methods of evaluation of accuracy of precise levelling were proposed. These methods are exactly described in the work (JORDAN et al. 1956, p. 223–255). The requirements which the precise levelling has to fulfill were defined first time during the second Surveyor Assembly in Berlin in 1897, and next were specified in 1871. As the results of these considerations to the evaluation of the accuracy of height differences from precise levelling the following formula was proposed

$$m_1^2 = \frac{1}{4n_l} \sum \frac{\rho^2}{l} \tag{4}$$

where ρ is discrepancies between forward and backward levelling of a section, n_l is a number of sections and l is the length of a section in km.

In this formula accidental and systematic errors which affect the levelling measurements are considered simultaneously.

The same accuracy evaluation can be obtained from the discrepancy λ of a forward and backward levelling of a line from the formula

$$m_2^2 = \frac{1}{4n_L} \sum \frac{\lambda^2}{L} \tag{5}$$

where L is the length of a line in km and n_L is the number of a lines.

The next information about the accuracy of the levelling network is included in the loop misclosures. The mean error of levelling can be computed from the formula

$$m_3^2 = \frac{1}{n_F} \sum \frac{\varphi^2}{F} \tag{6}$$

where φ is the loop misclosures in millimeters and F is the length of a loop in km, and n_F is a number of the loops.

The mean errors of the levelling network considered above can be evaluated before the network adjustment. After adjustment the accuracy of levelling network can be evaluated from the residuals v_i

$$m_o^2 = \frac{1}{n - u} \sum p v^2 \tag{7}$$

where m_o is a mean error computed from residuals, n is the number of observations and u i is the number of unknown parameters.

To evaluate the accuracy of the network of precise levelling obtained from campaign IV 16 150 discrepancies ρ from the forward and backward levelling of a sections, 282 discrepancies from the forward and backward levelling of a lines and 133 loop misclosures were used. After applying the formulas (4), (5) and (6) it yields $m_1 = \pm 0.278$ mm/$\sqrt{\text{km}}$, $m_2 = \pm 0.518$ mm/$\sqrt{\text{km}}$ and $m_3 = \pm 0.826$ mm/$\sqrt{\text{km}}$.

The Fig. 3 illustrates the results of the accuracy evaluation of the four national campaigns of the precise levelling counted from the above mentioned formulas. Errors evaluation of the first, second and third campaign were taken from (*Katalog*... 1939) and from monograph (WYRZYKOWSKI 1988).

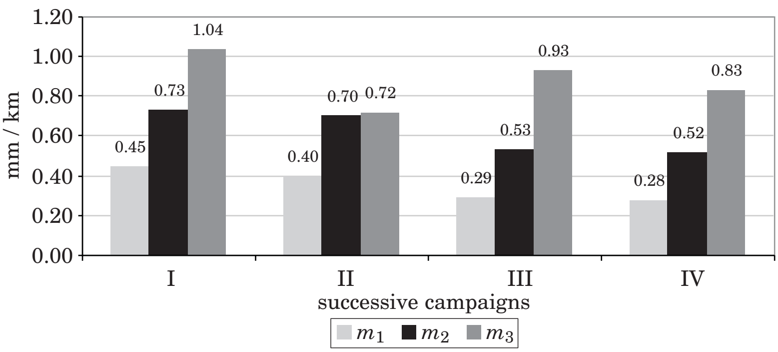

Fig. 3. Comparison of the errors m_1, m_2 and m_3 for the four successive levelling campaigns in Poland

From the Fig. 3 also results that the error m_1 is always smaller than the error m_2 and this subsequently is smaller then the error m_3. This regularity is fulfilled for all campaigns. This phenomenon shows the existence in the precise levelling one side acting factors which cause the growing of systematic errors. It means, that accidental and systematic errors should be considered separately in the proper way.

The value of errors m_1, m_2, and m_3 are getting more and more smaller in successive campaigns and it can be interpreted that instruments and methods used in the measurements are more and more exact.

For the better illustration of the accuracy evaluation of precise levelling some examples of the European levelling networks will be quoted beneath. In the case of Finnish levelling network measured in 1935–1955 we have $m_1 = \pm 0.33 \text{ mm}\sqrt{\text{km}}$, $m_2 = \pm 0.59 \text{ mm}\sqrt{\text{km}}$ and $m_3 = \pm 0.63 \text{ mm}\sqrt{\text{km}}$ (KÄÄRIÄINEN 1966). The Norwegian network measured in 1916–1953 can be characterized by $m_1 = \pm 0.84 \text{ mm}\sqrt{\text{km}}$, $m_2 = \pm 1.06 \text{ mm}\sqrt{\text{km}}$, (TROVAAG, JELSTRUP 1956), while the Dutch network has the accuracy: $m_1 = \pm 0.57 \text{ mm}\sqrt{\text{km}}$, $m_2 = \pm 0.88 \text{ mm}\sqrt{\text{km}}$ and $m_3 = \pm 1.1 \text{ mm}\sqrt{\text{km}}$ (NITTINGER, LUCHT 1971).

Lallemand formulas

The formulas for the accuracy estimation given the previous chapter, were subjected of the critical analysis on the International Conference of Surveyors in Hamburg in 1912 and finally formulas proposed by Lallemanda were accepted.

Lallemand assumed that the total mean error m of the levelling is a sum of random and systematic errors and can write down by the following formula.

$$m^2 = \eta^2 L + s^2 L^2 \tag{8}$$

where η is a random error, s is a systematic error and L is the length of a line in km.

According to the Lallemand the mean random error η should be computed from

$$\eta^2 = \frac{1}{4}\left[\frac{\Sigma \rho^2}{\Sigma L} - \frac{\Sigma l^2}{(\Sigma L)^2} \Sigma \frac{\lambda^2}{L}\right] \tag{9}$$

where ρ is the height difference of forward and backward section levelling, λ is the height difference of forward and backward line levelling and L is the length of the line.

Mean systematic error s is computed from

$$s^2 = \frac{1}{4\sum L} \Sigma \frac{\lambda^2}{L} \tag{10}$$

or using loop misclosures φ from the formula

$$s^2 = \frac{1}{\sum F^2}\left[\frac{1}{2}\Sigma \varphi^2 - \eta^2 \sum F\right] \tag{11}$$

where F is the length of the levelling loop.

Using the above Lallemand formulas the accuracy of the campaign IV was evaluated and for the mean random error the value $\eta = \pm 0.27$ mm/$\sqrt{\text{km}}$ was received. The mean systematic error computed from λ is $s = \pm 0.08$ mm/$\sqrt{\text{km}}$, while the same error computed from φ is $s = \pm 0.04$ mm/$\sqrt{\text{km}}$. The total error which is the combination of accidental and systematic error, equation (8), is $m = \pm 0.28$ mm/$\sqrt{\text{km}}$.

The results of accuracy evaluation of the all four campaigns of the precise levelling in Poland are presented in Table 2. The errors for first three campaigns were taken from the monograph (WYRZYKOWSKI 1988).

Table 2

Character of systematic and accidental errors of the four levelling campaigns computed by the Lallemand's formulas

Kind of error	Successive levelling campaigns			
	I 1926–1937	II 1952–1955	III 1974–1982	IV 1999–2003
Random error η	±0.46	±0.37	±0.28	±0.27
Systematic error s Estimated from λ	±0.08	±0.13	±0.08	±0.08
Systematic error s Estimated from φ	±0.08	±0.11	0.04	±0.03
Total error m	±0.47	0.40	0.29	±0.28

On the Fig. 4 the illustration of the Table 2 is presented.

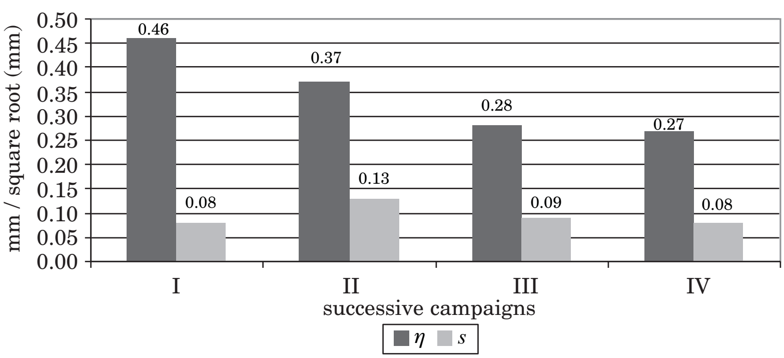

Fig. 4. The comparison of the errors η i s for the successive levelling campaigns in Poland

From the Fig. 4 result that the random error η in the successive campaigns decrease almost two times, from the value ± 0.46 mm/$\sqrt{\text{km}}$ to the value ± 0.27 mm/$\sqrt{\text{km}}$ while the systematic error remained on almost the same the level of 0.08 mm/km. Only for the campaign II the systematic error is somewhat larger and as for now it was not possible explain this phenomenon.

Getting smaller the value of the random error is evidences about using more and more exact instruments and measuring methods. On the other hand the value of the systematic errors in all campaigns is constant what can means that these errors reflect invariable for the Polish network such conditions like topography, climate (refraction) and movements of earth crust.

The comparison of the total error m counted according to the Lallemand's formulas with the error m_1 for individual campaigns is very interesting. From Fig. 3 and the last row of Table 2 results, that between these estimates, in principle, there are not significant differences. It authorizes to stating that these evaluations are equivalent.

Vignal formulas

Since the precise levelling observations are affected by the systematic one side acting errors, therefore at the Oslo Assembly of the IAG in 1948, the levelling error formulas were again reviewed and the new resolution for the method of estimation of the levelling was adopted.

The errors were divided into two groups, random and systematic group, which were assumed independent of each others. The random errors are caused by sources which are independent in all successive observations and obey Gauss's law of error distribution. The systematic errors are due to factors acting in the similar way on the successive or neighboring levelling observations. They do not obey Gauss's law. They become random only for distance exceeding a certain limit distance Z, which is a few tens of kilometers (see Fig. 5).

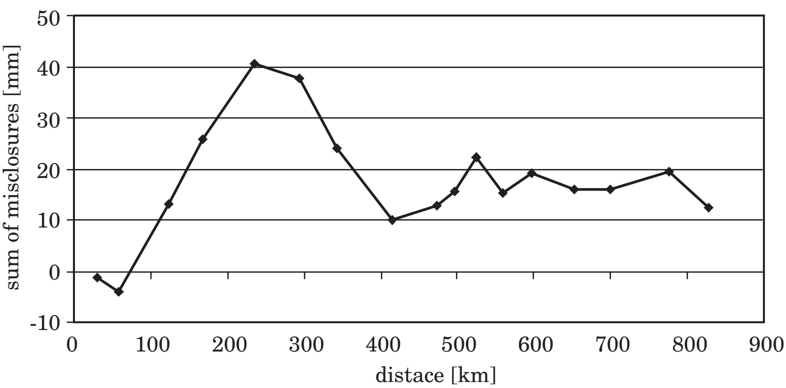

Fig. 5. Accumulation of misclosures λ for the line Szczecin-Białystok consist from 17 lines of precise levelling (see red line on Fig. 1)

From the graph shown on the Fig. 5 one clearly can saw that what the value of the sum of successive misclosures λ grows up to initially distance about 220 km what means, that the on the result of the measurement has influence the systematic factor, then the sum of misclosures begins oscillate around the value of 45 mm what means that systematic errors begin to keep as accidental errors. Below are given the basic information concerning Vignal's formulas.

The mean random error limiting value of the total error τ is the limit value of

$$u_L^2 = \frac{1}{4n_L} \Sigma \frac{\lambda^2}{L} \tag{12}$$

where λ is a line misclosures, L is a length of a line and n_L is a number of line in the network. In the case of the campaign IV the estimation of error u_L^2 has value 0.270 mm^2/km.

The mean random error is computed from

$$\eta^2 = u_r^2 - \zeta^2 \times j^2 \tag{13}$$

where u_r and j are computed from the formulas

$$u_r^2 = \frac{1}{4n_r} \Sigma \frac{\rho^2}{r} \quad \text{and} \quad j^2 = \frac{K}{Z} \times r_m \tag{14}$$

where ρ is a section misclosures, r is a length of a section and n_r is a number of the sections, r_m is a mean length of the section and Z, K are parameters. The value $K = 2$ and $Z = 50$ km were used here, since their values have no significance in this connection. In a case of campaign IV the error u_r^2 is 0.077 mm^2/km.

The mean random limiting value of the systematic error is consequently equal

$$\zeta^2 = \tau^2 - \eta^2 \tag{15}$$

where $\tau^2 = u_L^2$. According to these formulas the total error of the fourth campaign was computed in the following way

$$\tau^2 = u_L^2 = \frac{1}{4n_L} \Sigma \frac{\lambda^2}{L} = 0.27 \text{ mm}^2/\text{km} \tag{16}$$

$$\tau^2 = u_F^2 = \frac{1}{n_F + 1}\left(\sum \frac{\varphi^2}{F} + \frac{\varphi_e^2}{F_e}\right) = 0.69 \text{ mm}^2/\text{km} \tag{17}$$

To determine the systematic error ζ^2 and random error η^2 the system of equations should be solved

$$\begin{cases} \eta^2 = u_r^2 - \zeta^2 \cdot j^2 \\ \zeta^2 = u_L^2 - \eta^2 \end{cases} \tag{18}$$

The solution of this system of equations was realized by the method of successive approximations. In the first approximation we found, that $\eta^2 \approx u_r^2$ and we calculated the approximate value of the systematic error ζ. Then on the basis the approximate value the error ζ we calculated second time random error η and then the systematic error ζ. The final random and systematic errors were received after three iterations i.e. = 0.26 mm $/\sqrt{\text{km}}$ and ζ = 0.45 mm$/\sqrt{\text{km}}$. The total error containing the random and systematic part is $\tau = \pm\sqrt{\eta^2 + \zeta^2}$ 0.52 mm$/\sqrt{\text{km}}$.

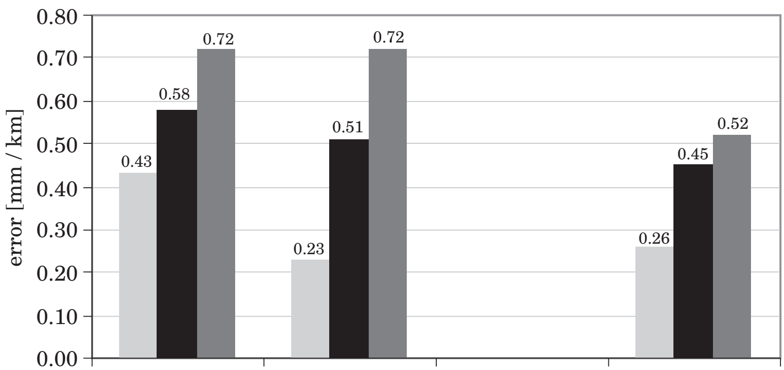

Fig. 6. Comparison of systematic and random errors computed from Vignal formulas for the successive levelling campaigns in Poland

On the Fig. 6 are presented values of the random error η, systematic error ζ and total error τ for the respective campaigns of the precise levelling in Poland. Because of the lack of data there are no evaluations for the campaign III.

From the Fig. 6 results that both the random and systematic error in the successive campaigns decrease, with the except of the random error from campaign II, which achieves exceptionally small value of ± 0.23 mm$/\sqrt{\text{km}}$. This evaluation was made by (WYRZYKOWSKI 1969) according to rather complicated and difficult to verifying calculations which could be incorrect.

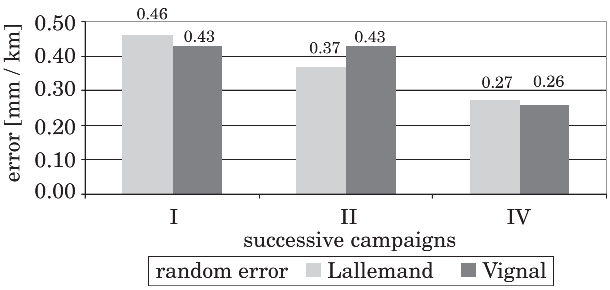

Fig. 7. Comparison of random errors computed from Lallemand and Vignal formulas for the successive levelling campaigns in Poland

On the Figs. 6 is introduced the comparison of the value of random errors counted from the Lallemand's and Vignal's formulas. It results that estimated errors from the Lallemand's formulas are somewhat larger than errors estimated from the Vignal's formulas in the case of the campaign I and II. However in the case of the campaign IV random errors computed in two different way are identical.

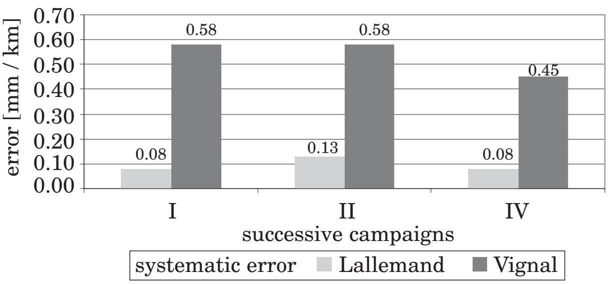

Fig. 8. Comparison of systematic errors computed by the Lallemand and Vignal formulas for the successive levelling campaigns in Poland

The comparison of systematic errors estimated from the Lallemand's and Vignal's formulas looks completely differently. From the Fig. 8 result that the systematic error computed from the Lallemand's formula is 0.08–0.13 mm/km, while the same error counted from the Vignal's formula gives four times larger value i.e. ±0.45 – ±0.58 mm/km. It is very difficult to say which evaluation is correct. One can affirm only, that such discrepancy proves difficulties when we trying to estimate systematic errors in the levelling networks.

Summary and conclusions

Exhausting analysis of the accuracy of the successive four campaigns of the precise levelling in Poland was described in the article. The evaluation of accuracy was conducted with utilization of traditional, Lallemand's and Vignal's formulas.

The results of the conducted evaluations confirm of the well known fact, that the error m_1 is always smaller than the error m_2 and it subsequently is smaller then the error m_3. (Fig. 3). This regularity is fulfilled for all campaigns. This phenomenon shows the existence in the precise levelling one side acting factors which cause the accumulation of systematic errors. It means, that accidental and systematic errors should be considered separately in the proper way.

The value of errors m_1, m_2, and m_3 are getting more and more smaller in the successive campaigns and it can be interpreted that instruments and methods used in the measurements are more and more precise.

The random error η in the successive campaigns decrease almost two times, from the value $\pm$ 0.46 mm/$\sqrt{\text{km}}$ to the value $\pm$0.27 mm/$\sqrt{\text{km}}$ while the systematic error remained on almost the same level of 0.08 mm/km. Getting smaller the value of the random error is evidences about using more and more precise instruments and measuring methods. On the other hand the value of the systematic error in all campaigns is constant what can means that this error reflects invariable for the Polish network conditions like topography, climate (refraction) and movements of earth crust.

Comparison of the total error m counted according to the Lallemand's formulas with the error m_1 for each campaigns shows not significant differences. It authorizes to stating that these evaluations are equivalent.

Assessment of the value of random errors counted from the Lallemand's and Vignal's formulas shows that estimated errors from the Lallemand's formulas are almost the same than errors estimated from the Vignal formulas. However systematic error computed from these both formulas gives quite different estimations and is very difficult to say which evaluation is correct.

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