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OPTIMAL LAND USE ANALYSIS USING LINDENMAYER GRAMMARS

***Urszula Żukowska*[1], *Grażyna Kowalewska*[2]**

[1] Department of Mathematical Methods of Informatics
University of Warmia and Mazury in Olsztyn
[2] Department of Quantitative Methods
University of Warmia and Mazury in Olsztyn

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Abstract

In today's world, when it is so important to use every piece of land for a particular purpose, both economically and ecologically, identifying optimal land use is a key issue. For this reason, an analysis of the optimal land use in a section of the city of Olsztyn, using the L-system Urban Development computer program, was chosen as the aim of this paper. The program uses the theories of L-systems and the cartographic method to obtain results in the form of sequences of productions or maps. For this reason, the first chapters outline both theories, i.e. the cartographic method to identify optimal land use and Lindenmayer grammars (called L-systems). An analysis based on a fragment of the map of Olsztyn was then carried out. Two functions were selected for the analysis: agricultural and forest-industrial. The results are presented as maps and sequences in individual steps.

ANALIZA OPTYMALNEGO UŻYTKOWANIA ZIEMI Z WYKORZYSTANIEM GRAMATYK LINDENMAYERA

***Urszula Żukowska*[1], *Grażyna Kowalewska*[2]**

[1] Katedra Metod Matematycznych Informatyki
Uniwersytet Warmińsko-Mazurski w Olsztynie
[2] Katedra Metod Ilościowych
Uniwersytet Warmińsko-Mazurski w Olsztynie

Słowa kluczowe: L-systemy, gramatyki Lindenmayera, optymalne użytkowanie ziemi, wartość ziemi, metoda kartograficzna.

Abstrakt

W dzisiejszym świecie, gdy tak ważne jest wykorzystanie ziemi w sposób celowy, ekonomiczny oraz zgodny z ekologią, rozwiązanie problemu optymalnego jej użytkowania wydaje się być zagadnieniem najważniejszym. Jako cel pracy obrano analizę optymalnego użytkowania ziemi na wycinku

Olsztyna, z wykorzystaniem programu komputerowego L-system Urban Delopment. Program wykorzystuje teorię L-systemów i metodę kartograficzną, pozwala na uzyskanie wyników w postaci sekwencji produkcji lub map. W pierwszej części przybliżono obydwie teorie, tj. metodę kartograficzną służącą do znajdowania optymalnego użytkowania ziemi oraz gramatyki Lindenmayera (zwane właśnie L-systemami). Następnie przeprowadzono analizę na podstawie fragmentu mapy Olsztyna. Do analizy wybrano dwie funkcje: rolną i leśną przemysłową. Wyniki zamieszczono na mapach oraz w postaci sekwencji w poszczególnych etapach.

Introduction

According to BAJEROWSKI'S (2003b) proposal, the state of land use may be understood as a function of the demand for the appropriate manner of using a given fragment of space. This fragment is a system which, because of its features, has one optimal state of use that "causes the highest land value from among the physically possible and legally permitted forms of use, consistent with its functional use" (BAJEROWSKI 2003b, p. 183). However, the existence of this optimal state of land use does not determine the actual manner of spatial use, which can be achieved by analysis and carrying out transforming measures. The following elements should be used to carry them out:

1. Selection of tools serving to find optimal land use.
2. Definition and examination of the parameters of the studied space which will allow transformation in the direction of optimal land use,
3. Selection of the method for estimating the profitability of undertaking the transformation (BAJEROWSKI 2003b).

Because space is represented by maps, the cartographic method seems most natural for its analysis. This method is proposed by BAJEROWSKI (1996, 2003b), who also notes that a spatial system is a dynamic system which can be analysed and forecast in the direction of optimal land use (BAJEROWSKI 2003a). However, this manner of using the cartographic method has some shortcomings, e.g. it becomes very painstaking and laborious without the use of supporting tools. For this reason, a combination of the cartographic method with Lindenmayer grammars has been proposed (ŻUKOWSKA 2008).

Lindenmayer grammars, also called L-systems or parallel rewriting systems, were originally created for description of the development of plant structures by the biologist Aristid Lindenmayer in 1968 (MARTYN 1996, PEITGEN et al. 1996, PRUSINKIEWICZ, LINDENMAYER 1996). However, the simplicity of their use and the possibility of extension with additional elements have allowed them to be used to simulate the development of not only individual elements of the plant or animal environment (PRUSINKIEWICZ 1993, PRUSINKIEWICZ et al. 1996, STREIT et al. 2005), but also landscapes and ecosystems (DEUSSEN et al. 1998). To present the results graphically, the so-called "turtle graphics", similar to the Logo language, were defined and simulation of the structure development in two and three dimensions was undertaken (PRUSINKIEWICZ et al. 1995, PEITGEN et al. 1996). Numerous computer programs using these grammars have been developed. One of them is the Virtual Laboratory (FEDERL, PRUSINKIEWICZ 1999), which can simulate plants with realistic shapes and also build other models, including models using fractal geometry. Another program is the Lsystem Urban Development, which combines the cartographic method proposed by Bajerowski and Lindenmayer grammars (ŻUKOWSKA 2012b). This program will serve here for analysis of optimal land use based on the example of a selected fragment of the map of Olsztyn. The program was made for finding the optimum usage of a land and combines two theories: modified Bajerowski's method and Lindenmayer's grammar. In this paper we try to answer to these question: Is modified method still working for finding the optimum usage of a land? Can we reach some vital conclusions from this application? Is this method suitable for economic use?

The application of the cartographic method for selection of the manner of optimal land use

According to SALICHCHEV (1998), cartographic research methods include, among others, mathematical modelling, which "consists in the creation of spatial mathematical models of phenomena or processes on the basis of data obtained from maps" (SALICHCHEV 1998, p. 272). For this purpose, Bajerowski uses topographic maps, land register content maps and the matrix of features inducing optimal land use. The method he proposes consists of the following stages:

1. A network of squares is superimposed on the selected map, whole or its fragment,
2. Selected features are then read from each field of the network and entered in the inventory matrix, the coordinates of matrix elements correspond to the coordinates of the corresponding field in the network,
3. The last stage is multiplication of the transposed matrix of features inducing optimal use and the inventory matrix, which gives as a result the matrix of optimal land use, which can be used for further analyses (BAJEROWSKI 1996, 2003b).

According to the first stage, a network of squares, i.e. basic fields, should be superimposed on the map. As can be read in Bartkowski, the basic evaluation field is the unit of area to which a specific geographical environment evaluation value can be assigned unambiguously (BARTKOWSKI 1974). The basic field does not have to be a square, but only this shape allows in the surface to be filled coherently and it can be easily compared with other fields. The size of the field so selected, e.g. for objects such as a village, should be from 4 to 30 ha (SENETRA, CIEŚLAK 2004). BAJEROWSKI (2000) is also in favour of selecting a square basic field, stressing that this is a practical shape for computer processing.

Reading and entering features from the map consists in using the digits "0" – feature absent and "1" – feature present. The inventory matrix of features is ready after this stage. The features referred to here can be read from the topographic map and from the land-in-use map.

The last stage is multiplication of the appropriate matrices. This stage and further analysis use the following manners of land use, also called planning land functions (BAJEROWSKI 1996, 2003a, 2003b): Agricultural: arable land (R), grassland (Ps), meadows (Ł); forest: productive (LsP), ecological (LsE), recreational: individual recreation (Wi), group recreation (Wz), without the right to build (Wn), community (B), infrastructural – industrial (P).

The above functions and expert methods allowed to create the *matrix of features inducing optimal land use* (BAJEROWSKI 1996, 2003a, 2003b). This matrix contains 56 features which can be read from the topographic map and the land-in-use map. The whole matrix can be found in the publications cited above. We will only mention here that the values from the table with plus or minus signs should be interpreted as the force with which a given feature influences the entry of the studied land function in the basic field. The points are added up, first positive, then negative, and finally both groups together. Their sum should give zero, which results from the fact that if all features occur, their effects cancel each other because it is not possible for one field to have all land functions simultaneously (BAJEROWSKI 2003b, p. 196).

The optimal land use matrix is obtained from multiplication of the appropriate matrices at the third stage. This matrix contains negative, zero and positive values. These values are interpreted appropriately. The first group, i.e. negative and zero values, should be discarded as useless for a given land function. As for the positive points, the maximum values should be found and used in further analysis.

Lindenmayer grammars

Lindenmayer grammars, or parallel rewriting systems, are a class of formal languages. The most basic groups of L-systems are: D0L-systems – deterministic context-free systems, IL-systems, context-sensitive systems, where I is the number of systems, stochastic systems, parametric L-systems (PRUSINKIEWICZ, LINDENMAYER 1996, PRUSINKIEWICZ et al. 1995, PRUSINKIEWICZ et al. 1996, PEITGEN et al. 1996). An important extension of L-systems is their enrichment with programming elements (PRUSINKIEWICZ et al. 1996).

The idea of L-systems consists in that we start with an axiom, which can be a single symbol or a set of symbols. Then the set of productions is reviewed in search of rules which fit a given symbol. If the rule is found, the symbol is deleted and the sequence in the rule is rewritten in its place. Sequences should be rewritten at the same time for all symbols. If the rule cannot be found for a given symbol, it is rewritten without changes. Theoretically the rewriting cycle can be continued ad infinitum. However, it is most often determined in advance how many times rewrites will be carried out. If needed, a rule can be included which will stop the whole process or which will cause only empty symbols, for which there will be no rules, to appear in the sequence and the whole process will stop (PEITGEN et al. 1996, PRUSINKIEWICZ, LINDENMAYER 1996).

The formal definition of a parametric context-free L-system will be presented below. A parametric OL-system is defined as an ordered quadruplet:

$$G = \langle V\,\Sigma,\ \omega\,P \rangle,$$

where: V – is the alphabet of the system, Σ – the set of formal parameters, $\omega \in (V \times \Re^*)^+$ is a non-empty parametric word called the axiom,

$$P \subset (V \times \Sigma^*) \times C(\Sigma) \times (V \times E(\Sigma))^*$$

is a finite set of productions.

The individual elements are understood as follows: $M^* = (V \times \Re^*)^*$ is the set of all module sequences, $M^+ = (V \times \Re^*)^+$ is the set of all non-empty module sequences, $C(\Sigma)$ is a logical expression, $E(\Sigma)$ is an arithmetic expression using parameters from the set Σ. We can use in logical and arithmetic expressions: parameters, numerical constants, arithmetic operators: +,–,*,/, exponentiation operator ^, relational operators: <,<=,>,>=,=,==, logical operators: !,&&, || (negation, and, or); brackets () and references to mathematical functions, e.g. sine, and to pseudo-random number generators (PRUSINKIEWICZ, LINDENMAYER 1996, PRUSINKIEWICZ et al. 1996).

A production in a parametric OL-system is denoted as $a : w \rightarrow \chi$, where: a – the predecessor, w – the condition and χ is the successor. A given symbol is replaced with the sequence χ if: the symbol equals a, the number of parameters is the same and the condition w is met. Moreover, we write $a \rightarrow \chi$ when the parametric word χ is successfully derived from the module a as a result of rewriting (PRUSINKIEWICZ, LINDENMAYER 1996, PRUSINKIEWICZ et al. 1996).

Compilation of the cartographic method and Lindenmayer grammars

The Lsystem Urban Development program uses the above-mentioned cartographic method. However, it was modified for the needs of the program and the use of Lindenmayer grammars. The first two stages are analogous to the above-described cartographic method. This means that a network of squares is superimposed on the prepared map. The features which can be found in the map are then written down in the inventory matrix. However, a change has taken place here compared to the described cartographic method. The features were grouped for more efficient reading, which is shown in Table 1.

This will give 8 inventory matrices, where the value “0” – that is the absence of a feature or the corresponding value from the table can appear in each matrix. The value of the dominant feature, as that most influencing the analysed basic field, is entered within a given group. This significantly shortens and simplifies the inventory of features. In the third stage, the product of the transposed matrix of features inducing optimal use and the inventory matrix is no longer conducted for the whole area but only for these fields which will be selected using the proposed Lindenmayer grammars.

The Lsystem Urban Development program stores the network of squares as a dynamic table of objects, where the object is a single basic field. This basic field contains some parameters read from the inventory matrix and parameters used for computations. Because it is impossible that the user can view the contents of the computer’s memory and thus check how the computations are conducted, a sequence containing all parameter values is displayed in an alphanumeric form. The definition of the module, which is identified with the basic field, is given below: SQ([no.x,no.y],d,f,v,w,z,s,dr,i,k,p,u), where individual parameters determine: no.x, no.y – x and y coordinates of a given square, d – direction of passing to the next field, f – land (use) function, v – sum of values for a given field, w – value from the waters group, z – value from the greenery group, s – value from the land structure group, dr – value from the roads and technical infrastructure group, i – value from the other land group, k – value from the land exposure group, p – value from the slopes group, u – value from the land in use group (ŻUKOWSKA 2008, 2012a, 2012b).

The direction of passing to the next field is selected at random. It is possible in the program to switch randomization: either the direction in each basic field is randomized once and does not change in subsequent steps or the direction in each basic field is randomized at each rewrite.

Table 1

Groups of features with letter designations

w – waters		*k* – land exposure:	
Value	Feature	Value	Feature
0	none	33	northern
1	lake shorelines	34	north-eastern
2	rivers and streams	35	eastern
3	canals and ditches	36	south-eastern
4	swamps and marshes	37	Southern
5	small standing waters	38	south-western
6	springs	39	western
13	wetlands	40	north-western
z – greenery		*p* – slopes	
0	none	41	0–3%
7	forest boundaries	42	3–6%
8	rows of trees	43	6–10%
9	groups of trees, groves	44	10–15%
10	single trees	45	15–25%
11	bush belts, hedges	46	25–35%
12	brushwood, bush clumps	47	Over 35%
s – land structure		*u* – land in use	
0	none	48	Meadows I-III
14	gorges, ravines	49	Meadows IV-V
15	scarps, embankments, excavations	50	Meadows VI
16	sands, boulder deposit areas	51	Grassland I-III
17	rocks, boulders	52	Grassland IV-V
		53	Grassland VI-VIz
		54	arable land I-IIIb
		55	arable land IVa-V
		56	arable land VI-Viz
i – other land		*dr* – roads and technical infrastructure	
0	none	0	none
18	devastated areas	22	power lines
19	industrial land in use	23	railway lines
20	buildings	24	hard-surfaced roads
21	ruins	25	improved roads
29	cemeteries and burial grounds	26	dirt roads
30	protected areas	27	Paths
31	natural monuments	28	Enclosures
32	historical monuments		

Source: ŻUKOWSKA (2008, 2012a, 2012b).

Selected rules used in the program are given below:
Example rule for direction selection:
SQ([no.x,no.y], d, f, v,w, z, s, dr, i, k, p, u) :
$d = 0 \rightarrow$ SQ([no.x -1,no.y], d, f, v,w, z, s, dr, i, k, p, u),
Example rule defining field usefulness:
SQ([no.x,no.y], d, f, v,w, z, s, dr, i, k, p, u) :
$V > 0 \wedge calc = 1 \rightarrow$ mark as useful for f,
Example rule defining the value for the agricultural function:
SQ([no.x,no.y], d, f, v,w, z, s, dr, i, k, R, u) :
$f = 1 \wedge w = 1 \wedge calc = 0 \rightarrow$ SQ([no.x,no.y], d, f, v-5,w, z, s, dr, i, k, p, u)

Optimal land use analysis using the Lsystem Urban Development program

To carry out an optimal land use analysis using the Lsystem Urban Development program, the user must inventory the features and enter them in inventory matrices in the program. The user must then select the land use function and select the coordinates of the initial basic field, or the axiom. The next step is to perform the rewriting. The results will be given as a table and the created sequence of productions. Additionally, if the user has a map, it is possible to present the results on the map, after prior specification of the map scale. Let us take a fragment of the map of Olsztyn as an example. An agricultural function – arable land is selected for the first analysis. The first axiom is a field with coordinates 2,2, the randomization direction will be selected in each rewrite:
Axiom: SQ([2,2],0,1,0,2,7,14,26,20,37,42,0);
First rewrite: SQ([2,2],6,1,14,2,7,14,26,20,37,42,0)
SQ([1,2],0,1,0,0,7,0,23,0,35,0,0);
Second rewrite: SQ([2,2],2,1,14,2,7,14,26,20,37,42,0)
SQ([1,2],5,1,-2,0,7,0,23,0,35,0,0) SQ([3,3],1,1,0,2,7,15,24,20,34,42,0);
Third rewrite: SQ([2,2],1,1,14,2,7,14,26,20,37,42,0)
SQ([1,2],2,1,-2,0,7,0,23,0,35,0,0) SQ([3,3],0,1,-3,2,7,15,24,20,34,42,0)
SQ([2,1],2,1,0,5,7,0,23,20,34,45,0) SQ([4,3],2,1,0,5,7,15,26,20,40,45,0).

The first axiom is a field with coordinates 2,2, the passing direction will be randomized only at field creation:
Axiom: SQ([2,2],6,1,0,2,7,14,26,20,37,42,0);
First rewrite: SQ([2,2],6,1,14,2,7,14,26,20,37,42,0)
SQ([3,3],0,1,0,2,7,15,24,20,34,42,0);
Second rewrite: SQ([2,2],6,1,14,2,7,14,26,20,37,42,0)
SQ([3,3],0,1,-3,2,7,15,24,20,34,42,0) SQ([2,3],2,1,0,2,7,14,26,0,40,42,0);
Third rewrite: SQ([2,2],6,1,14,2,7,14,26,20,37,42,0)
SQ([3,3],0,1,-3,2,7,15,24,20,34,42,0) SQ([2,3],2,1,13,2,7,14,26,0,40,42,0)

a) ***b)***

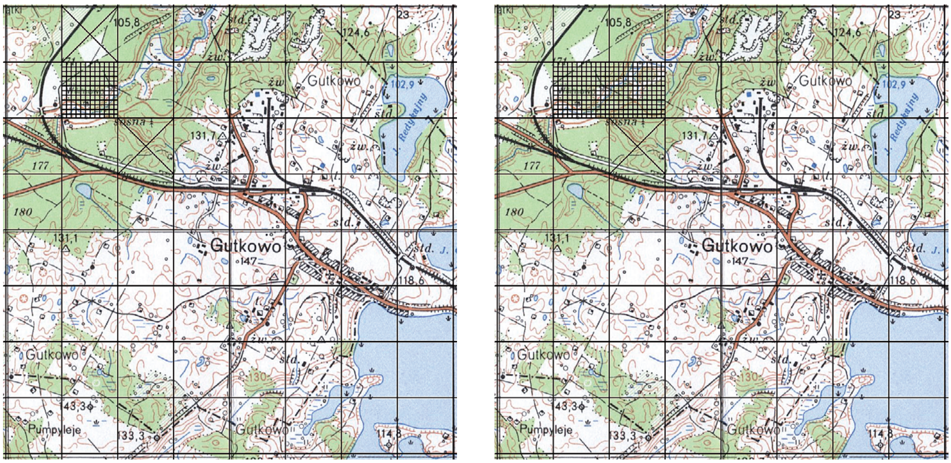

Fig. 1. Results: the squared fields denote areas suitable for the selected function, the crossed fields denote areas unsuitable for the selected function. The part (a) contains the result for the selected parameters when the direction of passing to the next square is randomized in each rewrite for each square and the part (b) the case when the direction is randomized once only at field creation
Source: Generated using the Lsystem Urban Development program.

An analysis itself of sequences created in successive rewrites already shows that the selected method of direction randomization when passing to the next field causes a higher or lower number of basic fields to be selected. If randomization in each rewrite is selected, basic fields can expand at an exponential rate. If the option that direction randomization is selected only once is chosen, then it may happen that at a certain point there are no longer new fields to be selected.

In Figure 1a and 1b the squared fields denote a field which is suitable for the selected function, in our case for arable land. The crossed fields denote that the basic fields are unsuitable for the selected function. It can be observed in both figures that the crossed fields are located within forests. This seems logical because the transformation cost for such land would be too high. Apart from legal aspects associated with changes in the local spatial management plan, aspects associated with land adaptation as well as ecological aspects, e.g. the effect of cutting down trees on the neighbouring areas and ecosystem, should be taken into account. Hence these areas are unsuitable for agricultural land. As can be seen, the selected method has confirmed its usefulness.

The fields with coordinates (2,2) and (2,3) were marked with a square pattern as those which are suitable for arable land. They are in a narrow, where there is a watercourse. Perhaps the value of these areas after transformation into arable land will exceed the transformation cost. However, it is interesting what happens when the same coordinates are selected, but a totally different land function (the forest productive function) is selected:

Forest industrial function, randomization of the direction of passing to new fields in each rewrite:
Axiom: SQ([2,2],1,4,0,2,7,14,26,20,37,42,0);
First rewrite: SQ([2,2],3,4,21,2,7,14,26,20,37,42,0)
SQ([3,2],6,4,0,0,7,15,24,20,33,0,0);
Second rewrite: SQ([2,2],6,4,21,2,7,14,26,20,37,42,0)
SQ([3,2],0,4,21,0,7,15,24,20,33,0,0) SQ([2,3],1,4,0,2,7,14,26,0,40,42,0)
SQ([4,3],4,4,0,5,7,15,26,20,40,45,0);
Third rewrite: SQ([2,2],7,4,21,2,7,14,26,20,37,42,0)
SQ([3,2],4,4,21,0,7,15,24,20,33,0,0) SQ([2,3],2,4,33,2,7,14,26,0,40,42,0)
SQ([4,3],1,4,22,5,7,15,26,20,40,45,0) SQ([3,3],4,4,0,2,7,15,24,20,34,42,0)
SQ([3,4],3,4,0,2,7,0,28,20,35,42,0)

Forest industrial function, direction randomization only once at field creation:
Axiom: SQ([2,2],1,4,0,2,7,14,26,20,37,42,0);
First rewrite: SQ([2,2],1,4,21,2,7,14,26,20,37,42,0)
SQ([3,2],4,4,0,0,7,15,24,20,33,0,0);
Second rewrite: SQ([2,2],1,4,21,2,7,14,26,20,37,42,0)
SQ([3,2],4,4,21,0,7,15,24,20,33,0,0) SQ([2,3],6,4,0,2,7,14,26,0,40,42,0);
Third rewrite: SQ([2,2],1,4,21,2,7,14,26,20,37,42,0)
SQ([3,2],4,4,21,0,7,15,24,20,33,0,0) SQ([2,3],6,4,33,2,7,14,26,0,40,42,0)
SQ([3,4],4,4,0,2,7,0,28,20,35,42,0)

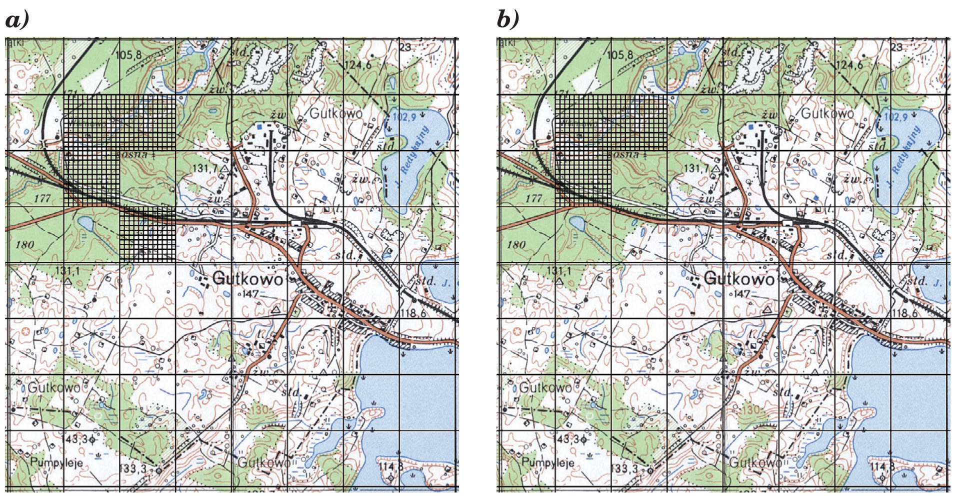

Fig. 2. Results for the forest industrial function. The squared fields are areas useful for the selected function. There are two cases: a) when the passing direction is randomized at each rewrite; b) when the passing direction is randomized at field creation

Analysing only the figures 2a and 2b themselves, it can be observed that the areas which were marked as favourable for the forest industrial function are fields dominated by forests. However, it would be worth checking the value v, which is the total value for individual fields and indicates the force with which a given function will be able to exist in a given field. For the agricultural function, it is 1 for the field with coordinates (2,2) and also 1 for the field with coordinates (2,3) and for the forest industrial function it is 4 for the fields (2,2) and (2,3). These two fields were not selected by chance – they were marked as favourable in both analyses. In the other cases, when a field is marked in one analysis as useless for a function, but is useful for another, the conclusion as to which function is more advantageous is obvious – the function is selected for which the field was marked as favourable. It would be different if fields were selected as favourable for both functions. The total value of the field, denoted as v, must be compared here and the one with the higher value selected. In the presented analysis, the forest productive function obtains the higher value for the fields (2,2) and (2,3), thus this function will bring more benefits for the considered transformation.

Recapitulation

The method presented in this study, a compilation of the cartographic method and Lindenmayer grammars, allows an optimal land use analysis to be easily performed. The results of this analysis can be used in spatial planning, in land value determination as well as in land transformation value determination. The presented example of an analysis demonstrates how this can be carried out and how the results can be interpreted. Because of the size of the paper, although it is not a full analysis, it nonetheless shows the capabilities of the discussed method. A full analysis would include comparison of all land use functions, and more rewrites could be conducted than only 3. Finally, in a full analysis, rewrites could be performed until all fields are filled. All of this depends on the needs of the conducted study.

As we described above the method is working for finding the optimum usage of a land and is working quickly and efficiently. The conclusions from analysis are simple and can be confirmed by pure analysis of map's fragment. Proposed method is also suitable for economic use especially for optimizing land use. Therefore this method is recommended for not only for people who works with urban or rural environment but also for economists, who are interested in finding optimum usage of a land. The specified analysis can be used for planning, changing purpose of a land and moreover for estimation of a land value in further analysis. It can be also a base for calculating rent for a land.

The selection of optimal land use is only one of the many applications of L-systems. The adopted solution is fast and simple, which is an unquestionable advantage in today's world. Further development of the method could involve inclusion of other basic field selection methods, perhaps using genetic algorithms and proposed program can be part of GIS.

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