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THE EFFECTIVENESS OF SIMPLE DIVERSIFICATION IN COMPARISON TO MARKOWITZ PORTFOLIO THEORY

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K e y w o r d s: simple diversification, portfolio effectiveness, systematic risk, specific risk.

A b s t r a c t

Analysis of capital investments at Warsaw Stock Exchange during the period of from 2004 through 2009 was the main goal of the study. The analysis was conducted on the base of the classic Markowitz portfolio theory and construction of multi-component balanced portfolios. The studies conducted indicate significant advantage of portfolio investments over the investments in individual securities. The largest risk decrease was recorded in case of the portfolios consisting of up to 5 securities.

Markowitz optimization applied in the studies allowed finding the portfolios much more secure, at the assumed rate of return, than the multi-component balanced portfolios. The results indicate significant importance of optimization models in the design of the portfolios of securities.

SKUTECZNOŚĆ DYWERSYFIKACJI PROSTEJ W PORÓWNANIU Z KLASYCZNYM MODELEM MARKOWITZA

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S ł o w a k l u c z o w e: dywersyfikacja prosta, efektywność portfela, ryzyko systematyczne, ryzyko specyficzne.

A b s t r a k t

Głównym celem pracy jest analiza ryzyka inwestycji kapitałowych na GPW w Warszawie od 2004 do 2009 roku. Analizę przeprowadzono na podstawie klasycznej teorii portfelowej Markowitza i budowy wieloskładnikowych portfeli równomiernych. Wyniki badań wskazują na znaczną przewagę inwestycji portfelowych nad inwestycjami w pojedyncze papiery wartościowe. Największy spadek ryzyka zaznaczył się dla portfeli składających się do pięciu walorów.

Wykorzystana w badaniu optymalizacja Markowitza pozwoliła na znalezienie portfeli znacznie bezpieczniejszych, przy założonej stopie zwrotu, od wieloelementowych portfeli równomiernych. Wyniki wskazują na niebagatelne znaczenie modeli optymalizacyjnych w konstrukcji portfeli papierów wartościowych.

Introduction

Risk in as inherent component of economic activity. It is visible particularly well in stock exchange investments where a large fluctuation of the rates of return occurs. The capital investments, in particular those in stocks, are characterized by significant risk resulting from numerous factors influencing the levels of quotes. Those factors can be independent of a given security and have a constant, systematic influence on the rates of return levels. This means they represent the so-called systematic non-diversifiable risk. The examples of variables that are systematic risk factors could be the market, inflation, interest rates, liquidity, macroeconomic and political changes risks (JAJUGA 1998). The other risk factors related to the activities of the individual issuers represent the source of the so-called specific, diversifiable risk. The investors, in building their investment portfolios take efforts to limit the risk in such a way that a decrease in prices for some stocks are balanced by increases in prices of other stocks. Construction of balanced portfolios containing equal shares of stocks of different enterprises represents the simplest method of diversification. According to the theory, with the increase in the number of assets in the portfolio its total risk decreases' this is confirmed by studies conducted at the Warsaw Stock Exchange (MARKOWSKI 2001). Construction of the effective portfolio, that is a portfolio with minimum risk for a given rate of return is a better method for risk elimination. The investors, however, for practical reasons, are building balanced portfolios, which are not the safest portfolios.

This study aimed at verifying to what extent investors building balanced portfolios expose themselves to excessive specific risk that could be diversified by applying the Markowitz model (1959). Analysis of the effects of simple diversification, that is the decrease of risk in balanced portfolios with expanding their composition by a larger number of stocks was another task of that study. The risk diversification study will be conducted considering sector membership of the companies used in the analysis.

Simple diversification versus optimal diversification

For the first time the bases of risk analysis in the form of the portfolio theory were presented by MARKOWITZ (1952). His theory marked the beginning of wide studies on capital investments and contributed immensely to formulation of the theory of investment choices and acquiring knowledge on the mechanisms determining the prices of financial instruments. The investment portfolio is an aggregated financial instrument consisting of individual instruments (e.g. stocks, bonds, treasury bills) forming that portfolio in the defined proportions. The following analysis encompasses the fundamental elements of the portfolio analysis and it will be limited to investing in risky instruments only.

Each instrument is characterized by the expected rate of return μ_i. The expected value of the rate of return for a given portfolio is the weighted average of the values of the expected rates of return for the individual instruments where the shares of each of the instruments in the entire portfolio represent the weights (COPELAND, WESTON 1979, pp. 146–147):

$$\mu_p = [\mu_1, \mu_2, \ldots, \mu_K] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} = \underline{\mu}^T \underline{X} \tag{1}$$

where μ_p – value of the expected portfolio rate of return; $\underline{\mu}^T$ – vector ($1\times K$) of the expected values for the individual instruments; $\underline{X}$ – vector ($K\times 1$) representing shares of instruments (x_i) in the portfolio; K – number of instruments in the portfolio.

According to the portfolio theory, the shares of individual securities in the investor's portfolio sum up to one, so:

$$\underline{X}^T \underline{I}_K = 1 \tag{2}$$

where $\underline{I}_K$ – vector ($K\times 1$) of ones, and in the market without possibilities of short sales the shares satisfy the inequality:

$$0 \le x_i \le 1;\ (i = 1, \ldots, K) \tag{3}$$

Individual securities (portfolios) may during a given period assume different values of the rate of return. The more those rates deviate from their average rate of return the higher is the risk of the given investment. The variance of the rate of return that is expressed by the square form that follows is the measure of risk that is the diversity of the achieved levels of the rate of return on a given investment (COPELAND, WESTON, 1979, p. 147):,

$$\sigma_P^2 = [x_1, x_2, \ldots, x_K] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \ldots & \sigma_{1K} \\ \sigma_{21} & \sigma_{22} & \ldots & \sigma_{2K} \\ . & . & . & . \\ . & . & . & . \\ \sigma_{1K} & \sigma_{2K} & \ldots & \sigma_{KK} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ \ \end{bmatrix} = \underline{X}^T \underline{\Omega} \underline{X} \tag{4}$$

where σ_P^2 – variance of the rate of return of the risky instruments portfolio; $\underline{\Omega}$ – positively determined matrix (K×K) of variances and covariances of the rates of return between the individual instruments.

The portfolio of capital investments may consist of one (single stock portfolio), a few, several or even, theoretically all the stocks available in the market. Each portfolio consisting of the sane stocks may contain them in different proportions. Minimization portfolio variance, that is decreasing the overall risk at the assumed level of the expected rate of return, is the goal of diversification.

In the portfolio theory context the nature of diversification can be presented in a simple way on the example of investments in the balanced portfolios, i.e. portfolios in which the shares of individual stocks in the portfolio are equal. The total portfolio risk expressed by the variance using formula (4) or in the scalar notation (ELTON, GRUBER 1998, pp. 72–73):

$$\sigma_P^2 = \sum_{i=1}^{K} x_i^2 \sigma_i^2 + \sum_{i=1}^{K} \sum_{\substack{j=1 \\ j \neq 1}}^{K} x_i \, x_j \, \sigma_{ij} \tag{6}$$

can be limited significantly. Accepting the assumption of identical weights of individual stocks in the portfolio the above formula can be written as follows:

$$\sigma_P^2 = \sum_{i=1}^{K} \frac{1}{K^2} \, \sigma_i^2 + \sum_{i=1}^{K} \sum_{\substack{j=1 \\ j \neq 1}}^{K} \frac{1}{K} \, \frac{1}{K} \, \sigma_{ij} \tag{6}$$

which after transformations is reduced to the format of:

$$\sigma_P^2 = \frac{1}{K} \, \overline{\sigma}_i^2 + \frac{K-1}{K} \, \overline{\sigma}_{ij} = \sigma_P^2 = \frac{1}{K} \, (\overline{\sigma}_i^2 - \overline{\sigma}_{ij}) + \overline{\sigma}_{ij} \tag{7}$$

where: σ_i^2 – average variance of the rates of return of companies in the portfolio; $\overline{\sigma}_{ij}$ – average covariance between the rates of return for the companies in the portfolio.

The above notation indicates that with the increase of the population K of the balanced portfolio, the first component of the portfolio variance aims at zero. That component represents the so-called diversifiable part of the total risk in the balanced portfolios that is the part that can be eliminated by combining individual securities into a portfolio. The second element of the balanced portfolio variance is the covariance risk that cannot be avoided in the simple diversification process. The risk of a very well diversified balanced portfolio will then aim at the level of the average covariance of the rates of return of the stocks included in the portfolio and it will express the level of the systematic risk. The influence of the number of securities in the portfolio on the level of that risk is presented in Figure 1.

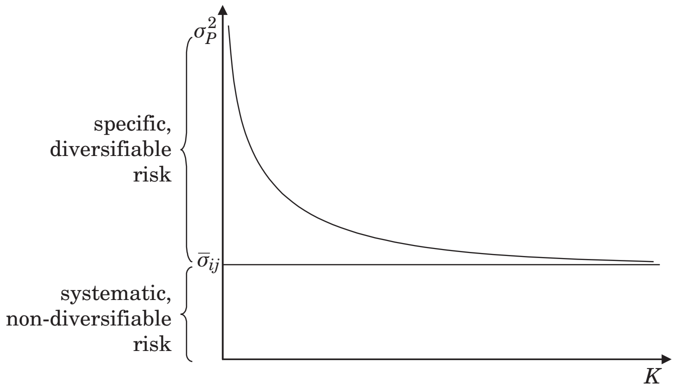

Fig. 1. Influence of the portfolio population on the risk size
Source: own work.

Balanced portfolios, even the most elaborate ones, may, but do not have to, represent investments assuring obtaining the minimum risk level at the required rate of return (effective portfolios). This results from the ineffective structure of the share of stocks in those portfolios, which make decreasing their total risk impossible.

In line with the above, simple diversification understood as expanding the size of the portfolio by adding new investments, will lead to limiting the risk, however, only skillful selection of the shares of stocks based on the covariances matrix analysis allows limiting the risk to the absolute minimum. The problem of determining portfolio weights minimizing the variance for the assumed level of the rate of return may be presented in the following way as an issue of the square programming:

$$\sigma_P^2 = \underline{X}^T \, \underline{\Omega} \, \underline{X} \to \min. \tag{8}$$

at given, linear limiting conditions:

$$\mu_P = \underline{\mu}^T \, \underline{X}, \qquad \underline{X}^T \, \underline{I}_k = 1 \tag{9}$$

The determined portfolios, minimizing the rate of return variance at the assumed level of the expected rate of return create the minimum risk set, which in the system of risk-revenue assumes the form of a parabola. The above situation is presented in Figure 2.

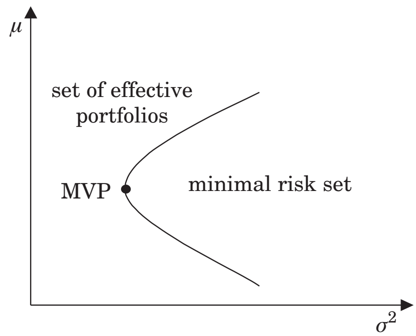

Fig. 2. Minimal risk set and the border of effective portfolios
Source: own work based on HAUGEN (1996).

The investors will consider only the portfolios situated on the upper arm of the parabola because only those generate the highest revenue at the approved risk level. Such portfolios, as a consequence, will form the set of effective portfolios. The MVP portfolio is the portfolio with the lowest risk that can be achieved.

Data

Information for the period of 2004–2009 was used for examining the level of capital investments; risk at Warsaw Stock Exchange. During the period covered 137 stocks belonging to the sectors of industry (79), finance (26) and services (32) were listed all the time. The time series on monthly rates of return were used in the studies.

The study applied the procedure of random generation of portfolios with the required population from among the initial set of the individual stocks[1]. To test the effects of simple diversification all the portfolios generated were balanced, i.e. the portfolio weights for each stock were equal to $x_i = 1/K$. In the study the portfolios with the population $K = 1, 2, 3, 4, 5, 10, 15, 20, 25$ were generated. With the exception of the single-element portfolios the number of

[1] The authors of this paper used the software by Maria Blangiewicz, M.Sc. (University of Gdańsk, Faculty of Management) written in the GAUSS language.

which was equal to the number of stocks in a given sector, for the remaining K values 1000 portfolios were generated at random for each of them.

Generating portfolios with different and systematically increasing number of stocks in the portfolio allowed analysis of portfolios risk diversification effects. The diversification coefficients d_k used in the study were computed according to the formula (BOŁT, MIŁOBĘDZKI 1996):

$$d_k = \left(\frac{\overline{\sigma}^2_{P,K}}{\overline{\sigma}^2_{P,1}} \right) \cdot 100 \tag{10}$$

where $\overline{\sigma}^2_{P,K}$ – average variance of the rate of return for a balanced portfolio consisting of K stocks; $\overline{\sigma}^2_{P,1}$ – average variance of the rate of return for an individual stock.

The problem of determining the vector $\underline{X}$ of the shares of stocks in the effective portfolios for the assumed rate of return was solved using the Wolf algorithm programmed in the Delphi language (RUTKOWSKA-ZIARKO 2005).

Results

The analysis of simple diversification effects on the base of the achieved rates of return of all the stocks during the period of 2004–2009 is presented in Table 1.

Table 1

Average rate of return, variance and diversification index of balanced portfolio determined on the base of all the listed companies

K	$\overline{R}_{P,K}$	$\overline{\sigma}^2_{P,K}$	d_K [%]
1	2.26	414.26	100.00
2	2.27	240.19	57.98
3	2.26	179.98	43.45
4	2.27	151.09	36.47
5	2.27	133.73	32.28
10	2.26	98.74	23.83
15	2.26	87.15	21.04
20	2.27	81.27	19.62
25	2.27	77.38	18.67
Optimal portfolio	2.27	10.56	2.55

Source: own computations.

The computed value of variance for individual securities is 414,26%^{2} for the average rate of return at ca. 2,27%. The risk level is subject to a significant reduction when consecutive stocks are added to the portfolio. It can be seen that the greatest decrease of risk occurs for portfolios of up to 5 stocks, for which the variance decreases three times as compared to the individual stocks. The further increase in the portfolio population causes slower and slower decrease of the risk. Thanks to the effects of portfolios diversification over 80% of the total risk for the individual securities can be eliminated. Elimination of total risk in the form of the diversification curve is presented in Figure 3.

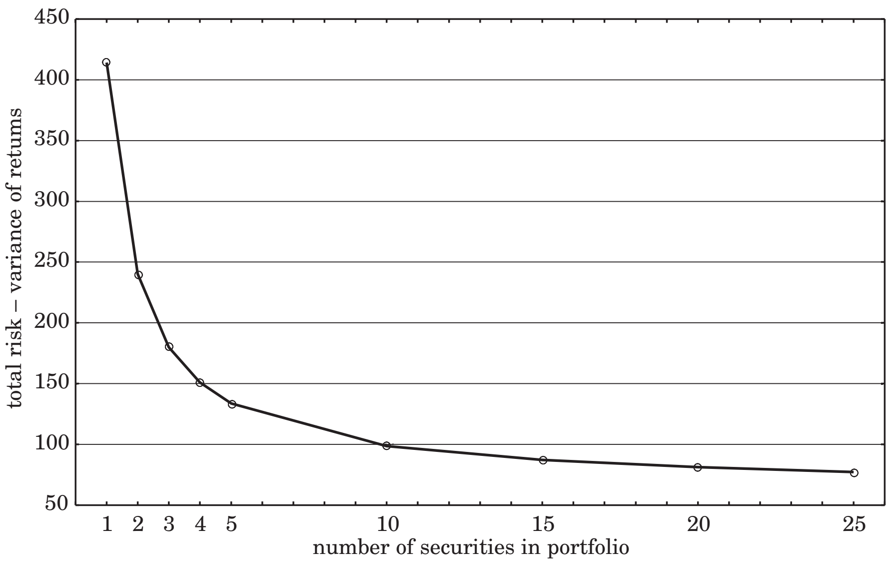

Fig. 3. Diversification curve for equally-weighted portfolios calculated basis of all securities quoted in period sample 2004–2009
Źródło: own calculations.

The optimization procedure for the assumed rate of return at 2.27% determined the effective portfolio with the variance of 10.56%^{2}. The Markowitz portfolio is characterized then by a few times lower risk than balanced portfolios of 25 elements. The effective portfolio consisted of the stocks of 25 different industrial companies. For 14 stocks their share in the portfolio was at least 0.01. Next the minimum risk portfolio was determined. The variance of that portfolio was 7.84%^{2} with the average profitability of 1.47%. That portfolio consisted of 20 different stocks although the shares of 5 of them were lower than 0.01.

Identical studies of simple and optimal diversification effects were conducted for the three main stock exchange sectors. The results are presented in Tables 2–4.

Table 2
Average rate of return, variance and diversification index of balanced portfolio determined on the base of industrial sector companies

K	$\overline{R}_{P,K}$	$\overline{\sigma}^2_{P,K}$	d_K [%]
1	2.35	414.40	100.00
2	2.35	237.54	57.32
3	2.35	178.70	43.12
4	2.35	150.37	36.29
5	2.35	132.00	31.85
10	2.35	97.64	23.56
15	2.35	85.58	20.65
20	2.35	79.82	19.26
25	2.35	75.22	18.15
Optimal portfolio	2.35	12.63	3.05

Source: own computations.

Table 3
Average rate of return, variance and diversification index of balanced portfolio determined on the base of finance sector companies

K	$\overline{R}_{P,K}$	$\overline{\sigma}^2_{P,K}$	d_K [%]
1	2.97	595.940	100.00
2	2.96	343.61	57.66
3	2.97	257.27	43.17
4	2.97	218.84	36.72
5	2.97	193.93	32.54
10	2.97	142.97	23.99
15	2.97	126.02	21.15
20	2.97	117.86	19.78
25	2.97	112.45	18.86
Optimal portfolio	2.97	45.81	7.68

Source: own computations.

Table 4
Average rate of return, variance and diversification index of balanced portfolio determined on the base of service sector companies

K	$\overline{R}_{P,K}$	$\overline{\sigma}^2_{P,K}$	d_K [%]
1	1.49	266.30	100.00
2	1.49	161.14	60.51
3	1.49	126.12	47.36
4	1.49	107.95	40.54
5	1.49	97.51	36.62
10	1.49	76.45	28.71
15	1.49	69.53	26.11
20	1.49	66.01	24.79
25	1.49	64.12	24.08
Optimal portfolio	1.49	25.93	9.79

Source: own computations.

The effects of simple diversification conducted for the companies from the sectors of industry, finance and services do not differ fundamentally from the results obtained in case of the entire population. The results of Markowitz portfolio analysis in the individual sectors were as follows:

– the optimal portfolio variance for the assumed rate of return at 2.35%, achieved in the sector of industry was $12.63\%^2$ and represented 3.05% of the average variance for the individual stock. The effective portfolio consisted of stocks of 20 different industrial enterprises. In case of 12 stocks, their share in the portfolio was at least 1%. The minimum risk portfolio for the industrial sector companies was characterized by the average profitability at the level of 1.61% and variance of $10.3\%^2$. It contained stocks of 19 enterprises and 13 of them had the share of over 0.01.

– the Markowitz portfolio variance for the assumed rate of return at 2.97%, achieved in the financial sector was $45.81\%^2$ and represented 7.68% of the average variance for the individual stock. The effective portfolio consisted of stocks of 12 different financial institutions; all shares were higher than 1%. The minimum risk portfolio was characterized by the average profitability at the level of 1.75% and variance of $29.6\%^2$. It contained stocks of 10 different financial institutions. For 9 companies the shares were higher than 0.01.

– the effective portfolio variance for the assumed rate of return at 1.49%, achieved in the sector of services was $25.93\%^2$ and represented 9.79% of the average variance for the individual stock. The effective portfolio consisted of stocks of 12 different service enterprises. Only one share was lower than 1%. The minimum risk portfolio built of service sector companies 49%, contained 12 components of which only one was lower than 0.01. The risk of that portfolio measured with the variance was 23.92%^{2} at the average rate of return at 1.11%.

Conclusions

The conducted studies indicate a significant advantage of portfolio investments over investments in individual securities. With the increase of the portfolio population the total risk expressed by the variance of the rate of return decreased, however, the largest decrease of the risk was observed in the portfolios consisting of up to 5 stocks. Further expanding the portfolio lead to much lower relative decreases of the risk.

The specific risk characterizing the individual securities is decreased by combining stocks into the larger and larger portfolios. The systematic risk, on the other hand, is the element that cannot be eliminated through diversification. It reflects the common reactions of the stocks to the external factors indifferent of the financial situation of the individual securities.

Markowitz optimization used in the studies allowed finding portfolios much safer than multi-component balanced portfolios. The results showed that portfolio diversification represents, first of all, skilful management of the set of securities available in the market, that is designing portfolios offering the possibly highest rate of return at the approved risk level as opposed to building portfolios based on the balanced expanding them by consecutive securities.

The work can be an indication for the investors, who should aim at limiting the risk by including stocks of many different companies into the portfolio. However, the best effects can be achieved by building the effective portfolios applying mathematical optimization models and appropriate software.

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