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THE ANALYSIS OF A TOTAL AND SYSTEMATIC RISK IN THE CONTEXT OF A DOWNSIDE RISK BASED ON THE EXAMPLE OF CAPITAL INVESTMENTS AT WARSAW STOCK EXCHANGE

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Key words: asymmetric measures of risk, downside risk, downside beta coefficients.

Abstract

Investments in large, medium and small companies listed at Warsaw Stock Exchange in the aspect of the downside risk were the major subject of the studies. For the analyzed companies, in addition to the variances and classic beta coefficients their downside equivalents, i.e. semivariances and semi-betas were determined. It was shown that companies of different size are characterized by the different levels of total and systematic risk. Additionally, semi-betas, being the measures of the downside systematic risk, are much stronger correlated with the profitability achieved than their classical equivalents.

ANALIZA RYZYKA CAŁKOWITEGO I SYSTEMATYCZNEGO W UJĘCIU DOLNOSTRONNYM INWESTYCJI KAPITAŁOWYCH W AKCJE SPÓŁEK NOTOWANYCH NA GPW W WARSZAWIE

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Słowa kluczowe: asymetryczne miary ryzyka, ryzyko dolnostronne, dolnostronne współczynniki beta.

Abstrakt

Głównym przedmiotem badań były inwestycje w duże, średnie i małe spółki notowane na GPW w Warszawie, w aspekcie ryzyka dolnostronnego. Dla analizowanych spółek wyznaczono, oprócz wariancji i klasycznych współczynników beta, ich dolnostronne odpowiedniki, tzn. semiwariancję i semibety. Wykazano, że spółki o różnej wielkości charakteryzują się odmiennym poziomem ryzyka całkowitego i systematycznego. Ponadto semibety, będące miarami dolnostronnego ryzyka systematycznego, są znacznie silniej skorelowane z osiąganymi rentownościami niż ich klasyczne odpowiedniki.

Introduction

The portfolio theory and valuation of securities according to the classical market equilibrium models, in particular the CAPM model, are based mainly on the assumption of normal distribution of the rates of return on securities and treatment of variance as the basic risk measure. While determination of distribution normality is, in most cases, subject to empirical verification, the assumption of variance, as the only appropriate risk measure seems to contradict intuition. According to the variance, the investors treat very high and very low rates of return as equally undesired. In reality, in line with rational decision taking, only the negative deviations are undesired as the positive ones create opportunities for high profit achievement. The negative attitude of investors concerning the rates of return lover than the level assumed causes that the asymmetric measures of systematic risk, in particular the measures of the downside risk should be treated as the appropriate measures of that risk. The left-sided risk perception allows repealing the assumption on normality of the rates of return distribution. The investors will prefer stocks with the lower downside level of systematic risk.

According to the above, the variance ceases to be the appropriate measure of the risk, while the measure reflecting the downside risk becomes desired. The semivariance, which is the average of the deviations below a defined level (MARKOWITZ 1959), is the basic measure for the negative deviations. Semivariance measures the downside variance only and in that sense it is believed to be a better risk measure than the variance. Semivariance is the so-called lower partial moment-lpm of the second order of the distribution of rates of return. The lower partial moments in approximation of downside risk are also reflected in the design of systematic risk measures such as the beta coefficient. The consequence of that are the downside beta coefficients that are of major importance in measurement and pricing of the capital investments risk (BAWA, LINDENBERG 1977, ESTRADA 2007, FISHBURN 1977, RUTKOWSKA-ZIARKO, MARKOWSKI 2009).

The paper aimed at the analysis of total and systematic risk, in particular in the aspect of downside risk, of capital assets listed at Warsaw Stock Exchange. The risk analysis was conducted for the companies included in the indexes representing the segments of large, medium and small enterprises.

Total and systematic risk according to the downside approach

Variance is the classic total risk measure in the finance theory. For the first time that statistical measure of dispersion was used for risk measurement by MARKOWITZ (1952). In practice, the value of variance is estimated on the base of empirical time series of the rates of return, the higher was the past variance of profitability of a certain stock the more risky it is considered:

$$s_i^2 = \frac{\sum_{t=1}^{m} (z_{it} - \bar{z}_i)^2}{m - 1} \tag{1}$$

where:

z_{it} – rate of return during period t for the stock exchange listed company i,
m – number of time units,
$\bar{z}_i$ – average rate of return for the stock exchange listed company i, estimated on the base of the historical data:

$$\bar{z}_i = \frac{1}{m} \sum_{t=1}^{m} z_{it} \tag{2}$$

The same treatment of negative and positive deviations from the average rate of return is the fundamental defect of variance as risk measure. In reality, negative deviations are undesired while the positive ones create opportunities for higher profit. Markowitz proposed semivariance, which is the average of deviations below the defined level for measurement of the negative deviations only (MARKOWITZ 1959).

$$ds_i^2\,(l) = \frac{\sum_{t=1}^{m} d_{it}^2\,(l)}{m - 1} \tag{3}$$

where:

$$d_{it}\,(l) = \begin{cases} 0 & \text{dla } z_{it} \geq l \\ z_{it} - l & \text{dla } z_{it} < l \end{cases} \tag{4}$$

$ds_i^2\,(l)$ – semivariance for the stock exchange listed company i,
l – equal to the average rate of return or the rate of return proposed by the investor.

The rate proposed by the investor may be a risk-free rate changing from period to period. Then we will receive the following formula for semivariance for the risk-free rate of return:

$$ds_i^2\,(f) = \frac{\sum_{t=1}^{m} d_{it}^2\,(f)}{m-1} \tag{5}$$

where:

$$d_{it}\,(l) = \begin{cases} 0 & \text{dla } z_{it} \geq z_{ft} \\ z_{it} - z_{ft} & \text{dla } z_{it} < z_{ft} \end{cases} \tag{6}$$

z_{ft} – risk-free rate of return during the period t.

Defining of the lower partial moments by BAWA (1975) and FISHBURN (1977) represented elaboration and generalization of semivariance as a risk measure. According to those authors the following expression is called the lower partial moment of degree n for stock i:

$$\text{LPMU}_i^n = \frac{1}{m} \sum_{t=1}^{m} lpm_{it}^n \tag{7}$$

where:

$$lpm_{it} = \begin{cases} 0 & \text{dla } z_{it} \geq l \\ z_{it} - l & \text{dla } z_{it} < l \end{cases} \tag{8}$$

Let us notice that for the lower partial moment is equal to the semivariance. The higher is the value of n the higher is the weight of high deviations below the assumed degree in the total value of the downside total risk. The level of the lower partial moment is related to the aversion of the investor to the risk, the higher the degree the higher is the aversion to the risk. The issue of the choice of the specific risk measure to a given investor or rather the utility function suitable for him becomes visible hear. In studies on the capital market that issue is generally disregarded and it is only assumed that the investor is characterized by aversion to risk and that he prefers higher rates of return to the lower ones. In that case semivariance, among others, can be the appropriate risk measure (MARKOWITZ 1959).

Application of classical beta (β_i) is linked to assuming the variance as the risk measure. Downside betas (β_i^{LPM}) on the other hand, are determined on the base of semivariance and other lower partial moments. In literature many types of lower betas have been identified dividing them according to the risk measure assumed and the reference point, which can be, e.g. the average, the risk-free rate or any assumed rate of return (ESTRADA 2007, KAPLANSKI 2004, GALAGEDERA, BROOKS 2007). Classical beta coefficients, as opposed to downside betas, assume one standard formula of regression coefficients in the Sharpe's model that has the form of:

$$z_{it} = \alpha_i + \beta_i z_{Mt} + \eta_{it} \tag{9}$$

where:

$$\beta_i = \frac{\mathrm{COV}_{iM}}{s_M^2} \tag{10}$$

z_{Mt} – market portfolio rate of return in the period t,
COV_{iM} – covariance of the rate of return for stock i and market portfolio rates of return,
s_M^2 – variance of market portfolio rates of return,
η_{it} – random component of the model.

In this study the assumption was made for determination of downside betas that the reference point is the risk-free rate changing its value from period to period (see: PRICE et al., 1982). Additionally the asymmetric mixed lower partial moment of second degree assuming the following format was used:

$$\mathrm{CLPM}_i^2 = \frac{1}{m} \sum_{t=1}^{m} (z_{it} - z_{ft})\, lpm_{Mt} \tag{11}$$

where:

$$lpm_{Mt} = \begin{cases} 0 & \text{dla } z_{Mt} \geq z_{ft} \\ z_{Mt} - z_{ft} & \text{dla } z_{Mt} < z_{ft} \end{cases} \tag{12}$$

where:
CLPM_i^2 – asymmetric mixed lower partial moment of second degree for stock exchange listed company i,
z_{ft} – risk-free rate of return during the period t.

The computation formula for the asymmetric mixed lower partial moment of second degree resembles the formula of classic covariance. It can be treated as the downside equivalent of that statistical measure. The value of the asymmetric mixed lower partial moment of second degree increases only when the rate of return for the stock and the market rate of return are simultaneously lower than the risk-free rate (see: HOGAN, WARREN 1974), which is presented in Table 1.

Table 1

Signs of the components of summing up in arithmetical computation of the asymmetric mixed lower partial moment of second degree depending on the market situation

Relation	$z_{Mt} < z_{ft}$	$z_{Mt} \geq z_{ft}$
$z_{it} < z_{ft}$	+	0
$z_{it} \geq z_{ft}$	–	0

Source: own work based on (HOGAN, WARREN 1977).

Considering (7) and (11), the downside betas determined according to the formula (see PRICE et al. 1982):

$$\beta_i^{\mathrm{LPM}} = \frac{\mathrm{CLPM}_i^2}{\mathrm{LPM}_M^2} = \frac{\mathrm{CLPM}_i^2}{ds_{\mathrm{M}}^2(f)} \tag{13}$$

where:

$ds_{\mathrm{M}}^2(f)$ – emivariance of the market portfolio determined in relation to the risk-free rate of return.

In case of the here presented approach, in determination of the downside beta coefficients the periods during which the market rate of return is higher than the risk-free rate of return are disregarded.

Results

The study encompassed companies listed at Warsaw Stock Exchange included in the indexes: WIG20, WIG40 and WIG80. The study was based on monthly rates of return for the analyzed stocks listed during the years 2000–2008. In total 59 companies listed at the stock exchange without interruption during the entire period covered by the study were analyzed. The companies were divided into three groups according to the size into large, medium and small companies. For each stock the average monthly rate of return was computed and according to increasing value of that parameter the companies were ranked within groups. For all the companies the variance, semivariance from the risk-free rate of return, classic beta coefficient and downside beta coefficient were computed. Also the difference between the betas ($\beta_i - \beta_i^{LPM}$) was determined, which represents the surplus of systematic double-sided risk above the downside systematic risk. The asymmetry coefficients (A) were computed and their significance for $\alpha = 0.05$ was tested. Significant asymmetry coefficients are presented in the following table in bold. The agreement of the distributions of rates of return for the analyzed companies with the normal distribution was tested by means of the Shapiro-Wilk test.

The results presented in Tables 2, 3 and 4 indicate that the majority of the companies studied are characterized by significant right-sided asymmetry. Only in case of seven companies consistency with normal distribution was recorded at the significance level of 0.05. In such a situation application of downside measures in risk analysis is justified.

During the period covered, the individual groups of companies were characterized by similar profitability, and the highest average rate of return was achieved by small companies. The differences between the average rates of return for the groups of large, medium and small companies were insignificant statistically. As concerns the total risk, it was the highest in case of small companies and the lowest in case of large ones. As concerns the systematic risk the opposite relation can be noticed that is, large companies showed stronger reaction to market changes while the small ones showed the weakest reaction. In case of large and medium companies the values of beta coefficients were, in average, higher than the values of semi-betas. This means, in general, that large and medium companies shower stronger reaction to changes in the stock exchange market during the periods of decrease as compared to the entire period. Small companies, on the other hand, react weaker to decreases in the market rate of return below the risk-free rate than to the fluctuations of the WIG index over the entire period. Considering the statistically the same level of the average rates of return, small companies are characterized by the highest level of the total risk and at the same time the lowest level of the systematic risk. The total risk can be decreased by appropriate selection of stocks for the portfolio while the systematic risk cannot be diversified and in that context investments in small companies are more attractive for the investor.

Further, the presence of correlation between the selected distribution parameters was tested using the Pearson's linear correlation coefficient (table 5). The significant coefficients ($\alpha = 0.05$) are presented in bold.

Table 2
Selected distribution parameters and risk measures for companies belonging to WIG20 index during the period of I 2000–XII 2008

Company	$\bar{z}_i$	s_i^2	$ds_i^2\,(f)$	A	β_i	β_i^{LPM}	$\beta_i - \beta_i^{LPM}$	S–W
AGO	-0.489	159.673	65.499	**1.728**	1.081	1.089	-0.008	
TPS	0.119	93.817	43.785	**0.851**	0.943	0.845	0.098	
PKN	0.357	74.301	39.215	0.035	0.953	0.974	-0.021	
ACP	0.835	247.416	105.380	**0.616**	1.364	1.266	0.098	
KGH	0.920	165.446	81.971	-0.235	1.360	1.328	0.032	
BRE	1.104	142.372	75.250	-0.453	1.233	1.299	-0.066	
PEO	1.171	77.072	35.249	0.019	1.011	0.977	0.034	consistent
CST	2.486	140.723	38.666	**1.163**	0.933	0.609	0.324	
PXM	2.594	248.423	74.567	**1.138**	1.249	1.023	0.226	consistent
PND	2.955	613.449	126.565	**2.246**	1.410	1.183	0.227	
In average	**1.205**	**196.269**	**68.615**	**0.711**	**1.154**	**1.059**	**0.094**	

Source: Own computations.

Table 3
Selected distribution parameters and risk measures for companies belonging to WIG40 index during the period of I 2000 – XII 2008

Company	$\bar{z}_i$	s_i^2	$ds_i^2\,(f)$	A	β_i	β_i^{LPM}	$\beta_i - \beta_i^{LPM}$	S–W
STX	-0.774	357.239	165.640	**0.856**	1.158	1.462	-0.304	
MIL	-0.551	165.387	95.331	0.039	1.355	1.592	-0.237	consistent
KRB	-0.100	76.870	46.031	-0.215	0.669	0.886	-0.217	consistent
BPH	0.073	165.673	126.721	**-3.375**	1.016	1.104	-0.088	
PGF	0.118	85.130	51.663	**-0.683**	0.616	0.844	-0.228	
BHW	0.153	75.083	43.107	-0.061	0.638	0.777	-0.140	
ORB	0.544	138.389	61.312	**0.585**	1.212	1.149	0.063	
KTY	0.629	113.486	53.587	**0.283**	0.867	0.885	-0.017	
BSK	0.737	70.250	36.065	-0.169	0.629	0.739	-0.110	consistent
MSZ	0.963	457.322	175.630	**0.808**	1.603	1.650	-0.047	
MSX	1.057	451.462	126.612	**2.449**	1.383	1.121	0.262	
ECH	1.094	195.972	104.198	**-0.707**	1.202	1.285	-0.083	
BDX	1.207	142.060	51.917	**0.688**	0.790	0.656	0.134	
GRJ	1.213	140.744	66.057	-0.015	0.744	0.647	0.097	
VST	1.556	275.102	85.398	**1.524**	0.805	0.715	0.090	
SNW	1.909	584.047	207.185	**0.790**	1.328	1.086	0.242	
ELB	1.936	122.756	36.432	**1.051**	0.736	0.554	0.182	
KPX	2.539	375.857	116.185	**1.086**	1.211	1.220	-0.010	
STP	4.453	246.348	58.447	**0.838**	0.801	0.499	0.302	
In average	**0.987**	**223.115**	**89.869**	**0.304**	**0.988**	**0.993**	**-0.006**	

Source: Own computations.

Table 4
Selected distribution parameters and risk measures for companies belonging to WIG80 index during the period of I 2000 – XII 2008

Company	$\bar{z}_i$	s_i^2	$ds_i^2\,(f)$	A	β_i	β_i^{LPM}	$\beta_i - \beta_i^{LPM}$	S–W
SWZ	-1.252	353.604	150.908	**1.489**	1.219	1.269	-0.050	
ADS	-1.050	179.508	89.715	**0.978**	0.693	0.734	-0.041	
PWK	-0.194	484.467	200.626	**1.112**	1.231	1.529	-0.298	
SGN	-0.010	186.559	86.298	**0.823**	0.773	0.804	-0.031	
IBS	0.223	522.077	180.203	**1.355**	1.236	1.180	0.057	
DBC	0.347	96.473	50.033	-0.295	0.643	0.839	-0.195	
BOS	0.352	54.243	23.245	**0.995**	0.299	0.316	-0.017	
LTX	0.354	243.769	95.431	**1.028**	0.991	0.942	0.048	
JPR	0.597	305.603	106.761	**1.690**	0.659	0.762	-0.103	
RFK	0.737	262.697	124.822	-0.105	1.001	1.029	-0.028	
MNI	0.860	508.195	196.794	**1.589**	1.240	1.153	0.087	
CMR	1.019	235.980	88.568	**1.016**	1.391	1.250	0.141	
SNK	1.102	150.716	50.962	**1.452**	0.756	0.727	0.029	
FCL	1.290	135.602	79.275	**-1.697**	0.693	0.794	-0.101	
PRC	1.396	1215.967	171.366	**5.611**	0.878	0.835	0.043	
KZS	1.406	556.828	163.216	**2.383**	0.726	0.864	-0.138	
MSC	1.419	123.073	61.625	-0.435	0.645	0.538	0.107	
PJP	1.474	185.330	61.731	**0.921**	0.845	0.674	0.171	
EPD	1.491	301.703	131.467	0.134	0.974	0.899	0.075	consistent
CSG	1.600	378.279	92.550	**1.992**	1.102	0.950	0.152	
IPX	1.974	304.748	114.593	0.450	1.343	1.194	0.149	consistent
MSW	2.027	246.116	95.798	**0.861**	0.455	0.381	0.074	
PGD	2.094	618.783	103.176	**3.682**	1.613	1.230	0.383	
ALM	2.350	244.882	80.991	**0.846**	1.007	0.669	0.338	
TIM	2.371	375.052	152.847	0.110	1.368	1.201	0.167	
BTM	2.377	407.603	136.554	**0.803**	0.827	0.612	0.215	
YWL	2.606	590.148	134.343	**1.816**	0.911	0.769	0.142	
APT	2.734	159.918	52.720	**0.557**	0.481	0.425	0.056	
ATS	2.852	1129.666	154.949	**3.766**	1.439	0.642	0.797	
BRS	2.997	389.407	95.354	**1.793**	1.406	1.141	0.266	
In average	**1.251**	**364.900**	**110.897**	**1.224**	**0.961**	**0.878**	**0.083**	

Source: Own computations.

Table 5
Pearson's linear correlation coefficients between selected distribution parameters for companies listed in WIG20, WIG40, WIG80 indexes during the period of I 2000 – XII 2008

Parameter	$\bar{z}_i$	s_i^2	$ds_i^2\,(f)$	A	β_i	β_i^{LPM}	$\beta_i - \beta_i^{LPM}$
$\bar{z}_i$	1.000	**0.318**	0.010	**0.261**	0.112	**-0.319**	**0.707**
s_i^2	**0.318**	1.000	**0.747**	**0.753**	**0.438**	0.179	**0.433**
$ds_i^2\,(f)$	0.010	**0.747**	1.000	**0.334**	**0.534**	**0.509**	0.053
A	**0.261**	**0.753**	**0.334**	1.000	0.236	-0.043	**0.463**
β_i	0.112	**0.438**	**0.534**	0.236	1.000	**0.816**	**0.323**
β_i^{LPM}	-0.319	0.179	**0.509**	-0.043	**0.816**	1.000	**-0.284**
$\beta_i - \beta_i^{LPM}$	0.707	0.433	0.053	**0.463**	**0.323**	**-0.284**	1.000

Source: Own computations.

The average profitability is correlated the strongest with the difference in betas. Significant correlations also exist between the average rate of return and the variance as well as asymmetry and downside beta coefficient. The fact of existence of significant correlation between the average and the variance coupled with lack of correlation with the classic beta is worth considering. No linear correlation was found between the semivariance and profitability but there is correlation between profitability and downside beta. It can be noticed that there is significant correlation between total risk measures and asymmetry. This means that asymmetry is an important aspect of investment at Warsaw Stock Exchange.

Conclusion

The studies conducted on the base of ten years monthly time series of rates of return for companies listed at Warsaw Stock Exchange show that the distributions of rates of return on investments in those companies very frequently diverge from the normal distribution. The study of downside risk, in case the assumption of normality of distributions of rates of return, is of major importance in managing (constructing) the securities portfolios.

Analysis of the risk of capital investments shows additionally the differences in its level for securities included in the indexes of small, medium and large companies. In case of statistically the same profitability level, large companies are characterized by the lowest level of the total risk while that risk is the highest in case of small companies. The level of systematic risk, in particular downside beta coefficients, which cannot be eliminated in the process of combining stocks into portfolios, is more important from the perspective of risk perception and diversification. The lowest values of that risk are achieved by small companies and in that context they seem the most attractive.

Significance tests of the linear correlation between selected parameters of distribution of rates of return showed existence of significant correlations between downside betas and the difference between betas with average rates of return as opposed to the lack of statistically significant correlation between the average rates of return and classic beta coefficients.

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