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MATHEMATICAL MODEL OF THE ROTOR SYSTEM OF A SUGAR CENTRIFUGE ACWW 1000

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K e y w o r d s: sugar centrifuge, centrifuge basket, mathematical model, equations of motion, Euler angles, gyroscopic phenomenon.

A b s t r a c t

The paper presents a mathematical model of the rotor system of a sugar centrifuge. The model was generated on the basis of a dynamic analysis of the gyroscopic system and equations of motion of the shaft and basket of the centrifuge. This model can be used for developing a model simulating the dynamic behavior of a centrifuge.

MODEL MATEMATYCZNY UKŁDAU WIRNIKOWEGO WIRÓWKI CUKROWNICZEJ ACWW 1000

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S ł o w a k l u c z o w e: wirówka cukrownicza, bęben wirówki, model matematyczny, równania ruchu, kąty Eulera, zjawisko żyroskopowe.

S t r e s z c z e n i e

Przedstawiono model matematyczny układu wirnikowego wirówki cukrowniczej ACWW 1000. Do budowy modelu wykorzystano analizę dynamiczną układu żyroskopowego, wyznaczając równania ruchu wału z bębnem rozpatrywanej wirówki. Model może być użyty do opracowania modelu wirówki symulującego jej zachowania dynamiczne.

Introduction and aim of the study

Experimental diagnostic studies of technical objects are a valuable source of information on their condition, but their scope is often limited due to the following factors (NIZIŃSKI, MICHALSKI 2002):

– it is not always possible to perform active experiments (especially in the case of large objects),
– the number of objects must be high in order to acquire sufficient and reliable information on sets of states and sets of parameters of diagnostic signals (symptoms of technical condition),
– the possibility of conducting experiments on single objects is limited,
– experimental studies are time-consuming,
– experimental studies are very expensive.

If the purpose of a given study cannot be attained due to the above limitations, experimental diagnostic investigations may be replaced by simulation tests (simulation experiments). Simulated models are generated on the basis of mathematical models. Depending on the system analyzed, mathematical models should be developed (if possible) on the basis of statistical, kinematic or dynamic analyses.

The objective of the present study was to generate a mathematical model of dynamic behavior of the rotor system of a sugar centrifuge ACWW 1000.

Object of the study

A sugar centrifuge, model ACWW1000, is used for centrifuging fillmass, to obtain second-grade sugar. This machine is used on average for two to three months a year. The centrifuge operates in a continuous mode – it is set in motion at the beginning of the sugar campaign, and stopped after its completion. If the operation is failure-free, the machine is not stopped during the campaign. Centrifuges belong to the group of critical machines due to the lack of parallel redundancy. In the case of failure, a centrifuge can be replaced by other machines fit for use, but it reduces production efficiency.

Technical characteristics:

– driving motor power: 55 kW;
– nominal revolutions of the driving motor: 1470 rpm.

The machine is fed by an inverter, which enables to achieve a higher rotational speed of the motor shaft. However, due to resonant vibrations of the centrifuge, the working rotational speed was set at 1800 rpm. The torque is directly transmitted from the motor to the shaft of the working basket by a flexible coupling. A scheme of the centrifuge is presented in Figure 1.

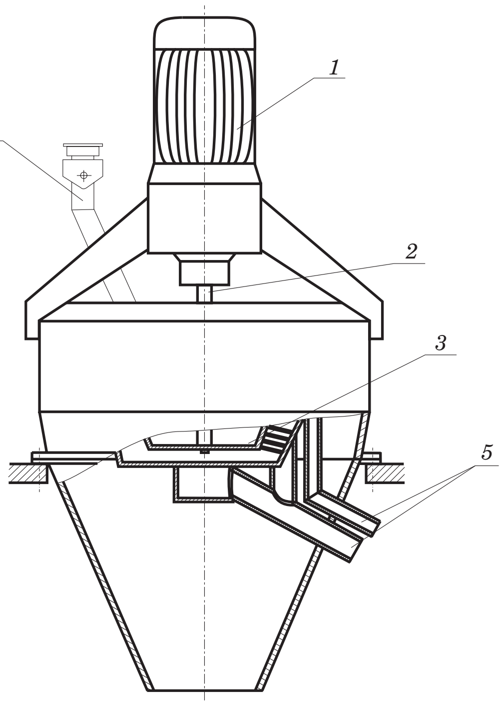

Fig. 1. Scheme of a sugar centrifuge, model ACWW 1000:
1 – motor, *2* – drive shaft, *3* – basket with sieves, *4* – feed pipe

Fillmass is transported to the basket (3) by the feed pipe (4), and uniformly distributed over the surface of the sieves under the influence of centrifugal force. As a result of centrifugation, fillmass is divided into powdered sugar and the so called run-off syrup on the sieves. These products are transported by the pipes (5). If fillmass is not prepared in the appropriate way, it may cause the occurrence of the so called technological unbalances, disturbing the dynamics of the rotor system.

Physical model

The power transmission system of a centrifuge ACWW 1000 is presented in Figure 2. It consists of an electric motor (1) connected with a shaft (3) by a flexible coupling (2). On the shaft (3) there is a basket (4) with sieves. The shaft with the basket is mounted unilaterally in two rolling bearings placed in the cradle of a gyroscopic seat. Details of mounting of the basket in bearings are given in Figure 3. The gyroscopic seat is screwed to the centrifuge body so as to ensure free mounting of the shaft with the basket.

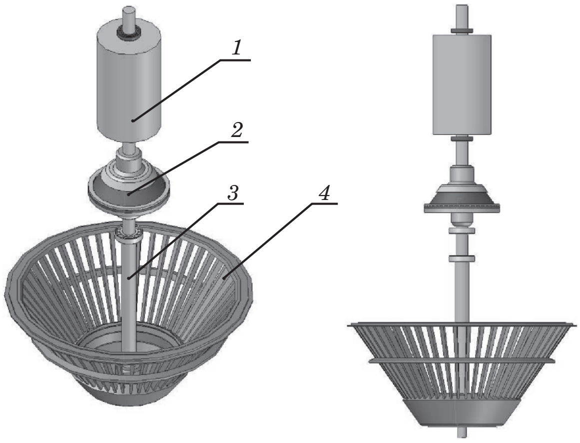

Fig. 2. View of the power transmission system of a sugar centrifuge, model ACWW 1000
1 – motor, *2* – flexible coupling, *3* – shaft, *4* – basket

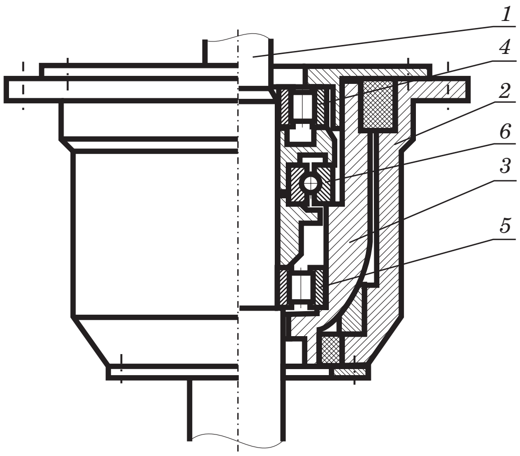

Fig. 3. Mounting of the shaft of a sugar centrifuge, model ACWW 1000, in bearings:
1 – shaft, *2* – gyroscopic seat, *3* – cradle, *4*, *5*, *6* – rolling bearings

The physical model shown in Figure 4 was taken for further considerations. In this model the shaft with the basket was treated as a rigid body with mass m. This is a gyroscopic system in which the center of mass (*SM*) of a rigid solid body is not the same as the center of rotation (*SO*) of this body. The distance between *SM* and *SO* is equal to l. The system is loaded by driving moment M and rotates at angular velocity ω, with respect to the axis of symmetry.

A coordinate system was determined according to generally accepted nomenclature (WOROSZYŁ 1976, LEYKO 2002) – Figure 4. The coordinate system *x,* *y, z* is a moving system, related to the rigid body analyzed, and the coordinate system x_1, y_1, z_1 is a fixed system, related to space. The origins of both systems are placed at the center of rotation of the rigid solid body. Euler angles, i.e. nutation angle θ, precession angle *y* and longitude of the nodal line φ (WOROSZYŁ 1976), were applied to describe the motion of the coordinate system.

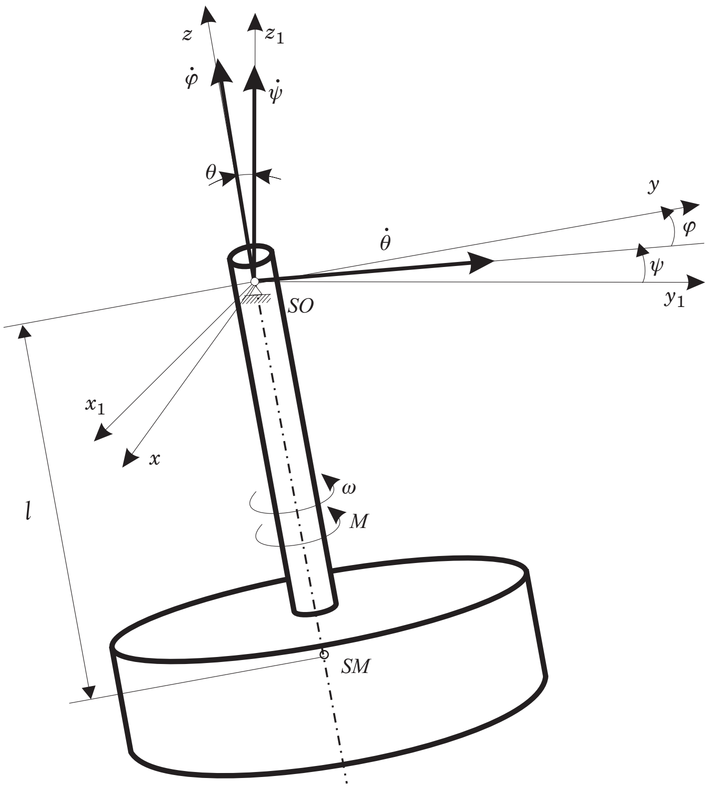

Fig. 4. Physical model of the system:
SO – center of rotation, *SM* – center of mass, l – distance between *SO* and *SM*, *M* – driving moment, ω – angular velocity, θ, φ, ψ – euler angles (nutation angle, longitude of the nodal line, precession angle $\dot{\theta},\dot{\varphi},\dot{\psi}$, – instantaneous angular velocities

Mathematical model

The following assumptions were made for the above physical model:

– mass passing through the centrifuge during one rotation is low, so changes in the mass of the system are considered in the dynamics of the system;
– the system is rigid enough to disregard displacements, velocities and accelerations of the center of rotation.

Angular velocities were determined in the moving system of coordinates (x, y, z):

$$\begin{cases} \omega_x = \dot{\theta}\sin\varphi - \dot{\psi}\sin\theta\cos\varphi \\ \omega_y = \dot{\theta}\cos\varphi + \dot{\psi}\sin\theta\sin\varphi \\ \omega_z = \dot{\varphi} + \dot{\psi}\cos\theta \end{cases} \tag{1}$$

where:

$\frac{d\varphi}{dt} = \dot{\varphi} = \omega = \text{const}$ (shaft rotation velocity)

The equations of motion of the system, presented as Lagrange equations, have the form:

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = M_\theta \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) - \frac{\partial T}{\partial \psi} + \frac{\partial V}{\partial \psi} = M_\psi \end{cases} \tag{2}$$

where:

M_θ – external moment of nutation,
M_ψ – external moment of precession,
V – potential energy of the system,
T – kinetic energy of the system.

Kinetic and potential energy of the system was determined on the basis of Equations 3 and 4.

The equation of kinetic energy has the form:

$$T = \frac{1}{2}\left(A\omega_x^2 + B\omega_y^2 + C\omega_z^2\right) \tag{3}$$

where:

A, B, C – principal moments of inertia of the system with respect to axes x, y, z.

Because the axis z is a symmetry axis of the system, $A = B$ and $C = J$.

The equation of potential energy has the form:

$$V = \frac{1}{2}\kappa\theta^2 \tag{4}$$

where:

κ – flexural rigidity of the shaft.

Substituting the expressions describing kinetic and potential energy to Lagrange equations, and considering the moment of nutation related to gravity force, $Mc = mgl\sin\theta$, we obtain equations of motion:

$$\begin{cases} A\ddot{\theta} + \dfrac{J-A}{2}\dot{\psi}^2 \sin 2\theta + J\omega\dot{\psi}\sin\theta + \kappa\theta = M_\theta - mgl\sin\theta \\ \left(A\sin^2\theta + J\cos^2\theta\right)\ddot{\psi} + \left(A\sin 2\theta - J\sin 2\theta\right)\dot{\psi}\dot{\theta} - J\omega\dot{\theta}\sin\theta = M_\psi \end{cases} \tag{5}$$

where:

ϖ – rotational speed,
A – principal moment of inertia with respect to axis x,
J – principal moment of inertia with respect to the symmetry axis,
κ – flexural rigidity of the shaft $\left(\kappa = \dfrac{EJ_w}{l_w}\right)$,
E – coefficient of direct elasticity (Young's modulus),
J_w – moment of inertia of the shaft section,
l_w – active length of the shaft,
M_θ – external moment of nutation,
M_ψ – external moment of precession,
m – rotating mass,
l – distance between the center of mass and the center of rotation,
g – acceleration of gravity.

The external moment of nutation represents moments forcing shaft deviation from the axis of the centrifuge. It includes moments of centrifugal forces related to unbalances of the basket. This moment can be contrasted with the moment M_c, related to gravity force. The external moment of precession represents non-uniformity of power transmission, caused among other by pulsation of the torque of an electric motor. In order to solve the system of equations (5) it is necessary to determine singular integrals, so numerical methods should be applied. The equations were solved by the Runge-Kutty method implemented in the computer program Mathcad (REGEL 2004) Initial conditions were determined on the basis of the experiment during which the trajectories of the motion of the shaft axis affected by known unbalance were recorded. Figure 5 illustrates the solution of the system of equations (5) in the form of curves of changes in the angle of nutation, nutation velocity and precession velocity in time.

An analysis of the curves shows that the frequency induced by precession of the system is equal to 9 Hz (according to the model). Figure 6 presents a spectrum of the velocity of vibrations and trajectories of the motion of the shaft axis recorded under operational conditions (during a sugar campaign).

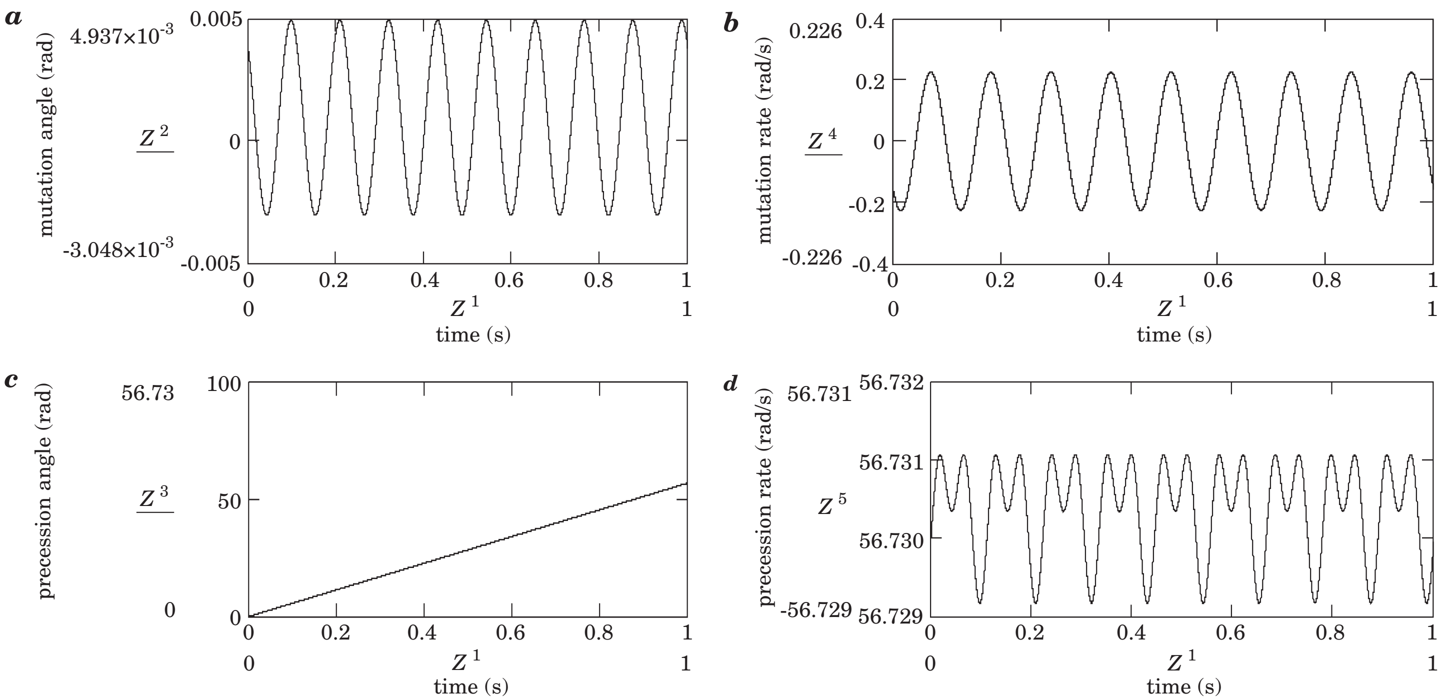

Fig. 5. Changes in the angle of nutation (a), angle of precession (b), velocity of nutation (c) and velocity of precession (d) of the centrifuge shaft

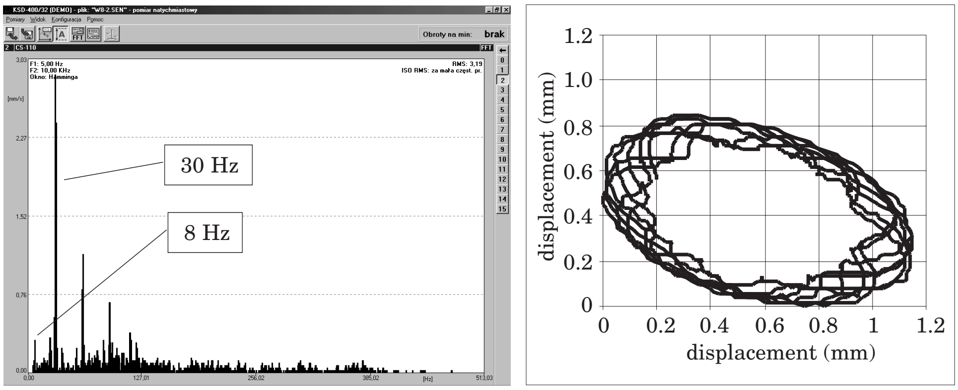

Fig. 6. Spectrum of the velocity of vibrations and the trajectory of the motion of the shaft axis recorded under operational conditions

There is a single line in the spectrum of vibration, corresponding to a frequency of 8 Hz, which indicates precession of the centrifuge shaft under the influence of unbalance. The increase in vibration amplitude for harmonic frequency I (30 Hz) suggests unbalance. The small difference (1 Hz) between the theoretical solution and results of experimental studies proves the correctness of both the model and solution.

Summary and Conclusions

The mathematical model of the rotor system of a sugar centrifuge, solved in the study, enabled to determine precession frequency. This in turn allowed to identify a spectral line corresponding to a frequency of about 8 Hz in the amplitude-frequency spectrum. Until now it was difficult to find the reason for the occurrence of vibrations at this frequency.

The mathematical model of the rotor system of a sugar centrifuge ACWW 1000, proposed in the study, describes the dynamic behavior of the system as affected by technological unbalances. This model provides the possibility to analyze changes in the angle of nutation in dependence on the unbalance of the rotor system. A negative effect is a deviation from the perpendicular of the shaft at an angle equal to the angle of nutation. Thus, simulation tests with the above mathematical model and active experiments with known unbalances, may provide the basis for determining boundary values of unbalance, considering the admissible angle of nutation.

Results of experimental investigations, comprising a spectral analysis and analysis of the trajectory of the motion of the shaft axis confirm the correctness of the mathematical model of the system examined in the study.

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