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SELECTED PROBLEMS OF FATIGUE OF MATERIALS AND CONSTRUCTION ELEMENTS

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Abstract

The study presents a survey of selected fatigue test methods and strength analyses of construction elements. The general characteristics of fatigue strength problems, with a focus on the procedure algorithms employed during strength assessment for the initiation of fatigue cracks and propagation life are given. The results of fatigue tests carried out on two different types of specimens – *CCT* and *SEN* – were compared.

WYBRANE ZAGADNIENIA ZMĘCZENIA MATERIAŁÓW I ELEMENTÓW KONSTRUKCJI

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Słowa kluczowe: zmęczenie materiałów, inicjacja i propagacja pęknięć, modele i prawa rozwoju pęknięć, próbki do badań zmęczeniowych.

Streszczenie

Przedstawiono przegląd wybranych metod badań zmęczeniowych i analiz oceny trwałości elementów konstrukcji. Podano ogólną charakterystykę zagadnień wytrzymałości zmęczeniowej, a szczególnie algorytmy postępowania przy ocenie trwałości do zainicjowania pęknięć zmęczeniowych i trwałości propagacyjnej. Porównano wyniki badań zmęczeniowych na dwóch rodzajach próbek – typu CCT i SEN.

Introduction

An extensive, chronological presentation of breakthrough events in the development of material and structure fatigue studies is included in (*Materialwissenschaft und...* 1993, SCHÜTZ 1996), whereas the most important problems of fatigue strength, fracture mechanics and material investigation are presented in (MANN 1978, KOCAŇDA 1985, SMITH 1986, SCHIJVE 1994, NEIMITZ 1996, PARIS 1998). The studies concentrate especially on the crack growth under service load conditions, delay effect tests of fatigue crack growth generated by overloads, and full-scale fatigue tests of aviation structures. The statistical nature of fatigue and crack growth characteristics require long-term investigations and a significant research effort to prepare their theoretical study before its implementation.

The last decades are marked by the development of a variety of methods directed towards the questions of structure fatigue reliability – especially as for the simulation of the fatigue crack initiation and propagation, non-destructive methods of state evaluation, and implementation of damage tolerant design concepts or probability risk estimations of service and boundary states (*Durability and damage...* 1985, *Impact damage...* 1975, YANG 1993). The problem of durability of construction elements is an interdisciplinary question – it combines numerous science domains, from mechanics, physics, mathematics and material engineering, through production technology, operational systems and logistics to (taking the most general option) philosophy, economy and politics. Considering the problem only in terms of mechanics and durability, it is still quite complex. Even in simple, standard cases, the subject requires an in-depth analysis of many issues including, among other, the determination of widely understood material and geometric characteristics of an element, the evaluation of a full-scale load spectrum, the estimation of durability on the basis of calculation models withelements of fracture mechanics, and empirical verification of the results obtained.

Strength Analysis Methods

Complex durability problems in design, production and service are solved with the use of the finite elements method, where finite elements are adapted to the needs of fracture mechanics and boundary integral equation methods (MUFTI et al. 1985, LEE, FENNER 1986, PUTCHKOV 1995). The determination of the material local stress state, the stress intensity factor or the damage degree generated by successive load cycles for complex three-dimensional construction elements are the main elements of the analysis of the above problems. Computational difficulties increase when crack growth is considered in fatigue life analyses not only at the stage of design, but also when solving certain operational problems.

In durability calculations, more and more often new fatigue criteria are taken into consideration (e.g. deformation and stress criteria for multiaxial cyclical loads (Gołoś 1987, Macha 1991, Neimitz 1998, Seweryn, Mróz 1996, Skibicki, Sempruch 1996, Szala 1998). A thorough analysis of the present state and development trends in fracture mechanics is presented in the papers by (Neimitz 1996, Neimitz 1998). Discussing the criteria of crack growth in brittle and plastic materials at different stages of their development, Neimitz (1996) pointed out to the discrepancies between theory and practice. It must be noted here that the question of nomenclature, vocabulary, notations and units connected with fracture mechanics has yet not been commonly acknowledged among engineers, though there are already some recognized standardizations and extensive studies dealing with the issue, e.g. (Francois 1996, Naumenko 1996).

A rapid development of fatigue crack growth models, especially under variable and random loads, has been observed in recent years. The methods for the estimation of fatigue crack growth rate can be classified according to different criteria. Depending on the mathematical apparatus used for the description of propagation, the models can be divided into deterministic and stochastic. Their practicability is verified in concrete applications. In general, however, it is not possible to talk about one universal model which could be applied to any type of material, environmental conditions, or different kinds of load cases. The most well-known models used for the evaluation of fatigue crack growth rate for variable loads, available via commercial software, are FASTRAN (Newman 1981, Newman 1984) and FASTRAN II (Newman 1992), a modified version of the former. In the analysis of fatigue crack growth in the materials under variable loads, two approaches may be distinguished: one uses mathematical statistics, and the other, deterministic, considers the influence of load sequence on fatigue crack growth. The former concerns mostly either different kinds of methods of substituting the variable load sequences for simplified equivalent loads, or the reduction of random variable load sequences to the sequences that can be described with known theoretical load distributions. The latter usually refers to the delay and acceleration effects of crack growth resulting from changes in load level, and is reflected in the formation of the so-called fatigue crack growth retardation models.

An intensive development of probabilistic methods aiming at the prediction of the fatigue crack effect progression, treated as a random process, has been observed recently. The dynamics of fatigue crack random growth is described with the use of Foker-Planck differential equations of the coefficients related to random distribution function parameters (e.g. Weibull, Poisson, logarithmic-normal). The solutions of these differential equations supply analytical forms for the probability density function of a random event in time, that is a crack of a given length. Thanks to the probability density function, it is possible to calculate the expected crack length, crack growth rate and fatigue life.

Operating load spectra include general overloads as well as underloads (or negative overloads), which cause a delay or acceleration of fatigue crack growth. A single tensile overload corresponds to an elementary case and the simplest situation leading to the crack propagation delay. Many authors have developed models describing crack growth under overload conditions on the basis of the correlation of the delayed crack growth transitory effects with different parameters connected with load, material qualities or characteristics of the external environment.

Despite the lack of one universal mechanism which would interpret all the characteristics referring to the fatigue crack growth delay, it is possible to identify basic mechanisms of the phenomenon. The said mechanisms are the following (SKORUPA 1996):

- compressing residual stresses in the overload plastic zone resulting from the material clamping action surrounding this zone,
- crack tip blunting that causes a decrease in the driving force in the crack tip,
- crack closure stemming from the contact of two crack surfaces above minimal loads, being a consequence of residual tensile strain in the material,
- crack tip strain hardening,
- change in the crack plane orientation as a result of the applied stress, and irregularities of the crack front shape, which can operate in the crack tip in such a way that they cause a crack growth delay.

The differences in the influence of plastic zones on the existence of a crack tip closure effect (or lack of this effect) in a plane stress state as well as in a plane strain state were described – with reference to aluminum and steel alloys – by MATSUOKA, TANAKA (1980). In aluminum alloys, a bigger proportion of the crack tip closure in the delay effect of its growth is observed on the specimen surface, whereas inside the specimen it is insignificant or does not occur at all (MCEVILY, YANG 1990). In the case of steel, on the contrary, the proportion of this effect is slightly higher when the distance from the specimen surface grows bigger. Various interpretations of the effect in question have been published. It was also found (SHUTER, GEARY 1995) that the delay after an overloading cycle does not have to demonstrate a direct correlation with the size of an overloading plastic zone. The cracks under delay conditions grow also outside the said zone. Stating at the same time that crack growth delay is greater near the surfaces of specimens than within them (thus it can be linked with the size of plastic zones), the authors concluded that the Wheeler's and Willenborg's models are good enough to describe the delay effect, and that they diminish the effect to the same degree when compared with that observed during experiments.

Other tests make use of the correlation between macroscopic propagation rate and the distances between the fatigue striations observed in fractographic tests (FLECK 1984, SHANIAWSKI, STEPANOV 1995, SIEGL, SCHIJVE 1990). Many researchers have considered this problem through the cycle-by-cycle analyses. They demonstrated that for different types of load and environmental conditions the crack growth, equal to the distance between the striations, is connected with a different number of cycles. To estimate the crack growth rate and durability, the line-elastic fracture mechanics assumes that the crack length increment corresponding to individual service load cycles is equal to the increments in the constant amplitude spectrum cycles characterized by the same amplitudes. The elasto-plastic fracture mechanics takes into consideration the influence of the load sequence on the crack growth increment in a given load cycle. Moreover, in the case of service analyses of load spectra and the durability of real objects, time is a very important factor. The occurrence of periods when the load is maintained on a constant level, characteristic, for example, of the operation of compressor disks as well as turbines and aircraft engine blades (SHANIAWSKI, STEPANOV 1995).

In the study (PARIS 1998), the author extensively analyzes a 40-year period over which fracture mechanics was applied to describe fatigue crack growth, pointing out to the cases when strength analysis was wrongly employed or obvious premises influencing the accuracy of durability estimation were ignored. These cases referred especially to the overuse of crack growth rules and crack shaping (such actions significantly affect strength calculations) as well as to ignoring in practice an important statistical output and other substantial considerations concerning the question. For instance, referring to the equation proposed in the dissertation, the author considers it to be one of the most frequently misused in strength analyses. During the crack growth, its rate suddenly increases, and the final durability estimation result depends crucially on the initial conditions, e.g. initial growth rate. Therefore, according to PARIS (1998), a precise estimation of local growth rates is extremely important to obtain accurate durability test results. In consequence, the application of the propagation equations, which means adjustment of a general crack growth trend rather than those local rates, is incorrect for durability estimations. The author (PARIS 1998) presented examples of fracture mechanics applications to the technical-construction issues (mostly those connected with aviation) and by disclosing their weak points, critically assessed some of the research trends developed during the last 40 years.

Methodology of durability estimation for the initiation and growth of fatigue cracks

To evaluate fatigue durability of construction elements, the following material characteristics are indispensable: material mechanical properties, material cyclic strain-stress curve, fatigue curves of the $\varepsilon = f(N)$ type (Manson-Coffin curves) or of the $\sigma = f(N)$ type (Wőhler curves) and/or $a = f(N)$ and $da/dN = f(\Delta K)$ propagation curves, where ε, σ denote strain and stress in a load cycle, a denotes crack length, N – the number of cycles, and ΔK – the range of stress intensity factor.

An elementary equation describing the behavior of metals in a low cycle fatigue range is an empirical relationship formulated by Manson and Coffin (KOCAŃDA, KOCAŃDA 1989, KOCAŃDA, SZALA 1991). It connects the number of cycles meant for destruction N_f with the plastic strain range $\Delta\varepsilon_{apl}$:

$$N_f^k \Delta\varepsilon_{apl} = C \tag{1}$$

where:

k, C – material constants.

For low carbon steel, low alloy steel and stainless austenitic steel, characterized by the strength stress of $R_m < 700$ MPa, the exponent $k \cong 0.5$.

The constant C, characterizing the plasticity level of steel, is determined from the following relationship:

$$C = \frac{1}{2} e_{rz} = \frac{1}{2} \ln \frac{100}{100 - Z} \tag{2}$$

where:

e_{rz} – real strain,
Z – reduction of the specimen area at a static rupture, expressed in %.

The Morrow's formula has the widest application in that area (KOCAŃDA, KOCAŃDA 1989):

$$\varepsilon_{ac} = \varepsilon_{as} + \varepsilon_{apl} = \frac{\sigma'_f}{E} \left(2N_f\right)^b + \varepsilon'_f \left(2N_f\right)^c \tag{3}$$

where:

σ'_f, b – coefficient and exponent of fatigue strength;
ε'_f, c – fatigue ductility coefficient and cyclic strain hardening exponent;
ε_{ac}, ε_{as}, ε_{apl} – total strain and its (elastic and plastic) components.

The Manson's equation (KOCAŃDA, KOCAŃDA 1989) has a similar structure to formula (3). It is based on the data from the static tension test, where the exponents $b = -0.12$ and $c = -0.6$ are assumed to be constant:

$$\Delta\varepsilon_{ac} = \Delta\varepsilon_{as} + \Delta\varepsilon_{apl} = 3.5 \frac{R_m}{E} \left(N_f\right)^{-0.12} + \varepsilon_{rz}^{0.6} \left(N_f\right)^{-0.6} \tag{4}$$

where:

R_m – tensile strength,
E – Young's modulus.

Another simplification of formula (3) useful in engineering calculations is the Langer's formula (MACHUTOV 1981):

$$\varepsilon_{ac} = \varepsilon_{apl} + \varepsilon_{as} = \frac{1}{4\left(N_f\right)^{0.5}} \ln \frac{100}{100 - Z} + \frac{Z_{-1}}{E} \tag{5}$$

The first term of equation (5) was generated from the second term of equation (3), where Langer assumed the constancy $\varepsilon'_f = 0.35\varepsilon'_{rz}$ and $c = -0.5$.

A low correlation between the stress amplitude sa and the number of cycles N_f made Langer exchange σ_a for the fatigue limit Z_{-1} in a symmetric cycle. The quantity Z_{-1} can be determined from the following relationship:

$$Z_{-1} = \gamma \cdot R_m \tag{6}$$

where:

γ – steel characteristics.

For $R_m \leq 700$ MPa the coefficient $\gamma = 0.4 \div 0.55$. In most cases it is assumed that $\gamma = 0.4$.

In the study by (Machutow 1987) the following formula was suggested for the calculation of a total strain:

$$\varepsilon_{ac} = \varepsilon_{apl} + \varepsilon_{as} = \frac{1}{4 \cdot \left(N_f\right)^{0.5}} \cdot \ln \frac{100}{100 - Z} + 0.435 \frac{R_u}{E \cdot N_f} \cdot k_s \tag{7}$$

where:

R_u – rupture stress,

k_s = 0.09÷0.12. Usually it is assumed that $k_s = 0.1$.

Formulas (3), (5) and (7) refer to the symmetric cycle with a rigid load. Following Machutov (1981), it is suggested that in the case of non-symmetric cycles, for which the plastic strain amplitude is lower than in a symmetric cycle, an average plastic strain amplitude should be used in formula (5):

$$\varepsilon_{apl,śr} = \varepsilon_{apl} \frac{1 + R_\varepsilon}{1 - R_\varepsilon} \tag{8}$$

$$R_\varepsilon = \frac{\varepsilon_{\min}}{\varepsilon_{\max}} \tag{9}$$

where:

$\varepsilon_{\min}$, $\varepsilon_{\max}$ – minimal and maximal strain in a cycle.

The decreasing of the fatigue limit in non-symmetric load cycles is considered in the second term of equation (5) by the introduction of the following relationship:

$$f\left(R_\varepsilon\right) = \frac{1}{1 + \frac{Z_{-1}}{R_m} \cdot \frac{1 + R_\varepsilon}{1 - R_\varepsilon}} \tag{10}$$

The Langer's formula, after considering relationships (8) and (10), assumes for the non-symmetric cycles the following form:

$$\varepsilon_{ac} = \frac{1}{4\left(N_f\right)^{0.5} + \frac{1+R_\varepsilon}{1-R_\varepsilon}} \cdot \ln\frac{100}{100-Z} + \frac{Z_{-1}}{E\left(1+\frac{Z_{-1}}{R_m}\cdot\frac{1+R_\varepsilon}{1-R_\varepsilon}\right)} \tag{11}$$

General notation of formula (11) can be expressed as:

$$\varepsilon_{ac} = \frac{1}{4}\varepsilon_{rz}\cdot f\left(N_f\right)\cdot f\left(R_\varepsilon, N_f\right) + \frac{Z_{-1}}{E}\cdot f\left(R_\varepsilon\right) \tag{12}$$

where:

$$f\left(R_\varepsilon, N_f\right) = \frac{1}{\left[1+0.25\cdot\left(\frac{1+R_\varepsilon}{1-R_\varepsilon}\right)N_f^{-0.5}\right]} \tag{13}$$

$$f\left(N_f\right) = \frac{1}{\left(N_f\right)^{0.5}} \tag{14}$$

According to equation (13), the effect of the asymmetry of a cycle on the value of the elastic-plastic strain amplitude within the range of cycle number $N = 5 \cdot 10^2$ to $N = 5 \cdot 10^5$ is insignificant.

The fatigue life estimated according to the above formulas can be burdened with a discrepancy in relation to a real service life. The durability expressed as the number of cycles until the fatigue crack initiation, and characteristic of a given load spectrum, can be, in fact, determined according to the procedure descried below. The procedure results in satisfactory estimations (Bukowski et al. 1992, Kłysz 1991, Kłysz 1999, Sobczykiewicz et al. 1983, Goss 2004):

– for the ranges $\Delta\sigma_i$ of successive load spectrum cycles (i is the number of cycles in a spectrum), determine the ranges $\Delta\varepsilon_i$ characteristic of a cyclic strain curve of the material tested, in the form of:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{2K}\right)^{1/n} \tag{15}$$

– for the ranges $\Delta\varepsilon_i$ of successive load spectrum cycles, determine the number of cycles for damaging $2N_{f,i}$ characteristic of the Manson-Coffin curve of the material tested, in the form of:

$$\frac{\Delta\varepsilon}{2} = \varepsilon_f \left(2N_f\right)^b + \frac{\sigma_f - \sigma_{śr}}{E}\left(2N_f\right)^c \tag{16}$$

– calculate the partial damages $D_i = \dfrac{1}{2N_{f,i}}$ generated by successive load spectrum cycles,
– calculate the total damage $D = \Sigma D_i$ characteristic of a given load spectrum,
– calculate durability (life) $N_I = 1/D$ for crack initiation, as an inverse of a total damage.

In the case of propagation life analyses, i.e. the analyses of fatigue crack growth up to critical values resulting in the damage of a structure, the procedure algorithm should include (BUKOWSKI et al. 1992, KŁYSZ 1999, KŁYSZ 2001, KŁYSZ 2002):
– determination a geometric configuration of a construction element with a crack of a given initial length which is usually assumed to be the minimal detectable one (e.g. with the use of non-destructive methods).
– assumption of a proper form of the shape function of the stress intensity factor K, corresponding to a given configuration of construction element, crack and load,
– determination of the material characteristics of the material analyzed, including propagation equation coefficients and possible ranges of their use, crack growth retardation model coefficients, the threshold value K_{th} and the critical value K_c of the stress intensity factor.

The computational procedure for each load spectrum cycle is the following:
– for given crack length values a and stress range $\Delta\sigma$ in a cycle, the value of the range of the stress intensity factor ΔK is determined;
– the crack length growth da in a given load cycle is calculated from a propagation equation;
– by applying the current load value to the preceding load sequence, it is found out whether there are conditions for a crack growth retardation resulting from an overloading cycle and, if necessary, the delay coefficient values are calculated according to the retardation model adopted;
– the determined crack length growth value, or the value reduced with the help of a retardation coefficient, is added to the current crack length.

For a new crack length value and a successive load cycle, the computational procedure is repeated until the critical value of the stress intensity factor K_c (fracture toughness) is obtained. This criterion indicates the damage of a construction element. The number of load cycles or spectra after which the value K_c was obtained, is the propagation life for the considered case of element geometry, the type of material used and the load applied in the study.

Examples of a model description of fatigue crack propagation

A general relationship of the rate of fatigue crack growth has the following form (O'DONOGHUE et al. 1995):

$$\frac{da}{dN} = \frac{A(1-R)^n (\Delta K)^m (\Delta K - \Delta K_{th})^p}{[(1-R)K_c - \Delta K]^q} \tag{17}$$

where the coefficient A and the exponents n, m, p, q are determined empirically.

More well-known relationships, such as the equations formulated by Paris (PARIS, ERDOGAN 1963), FORMAN (WHEELER 1970), or WALKER (WALKER 1970), are special cases of the above relationship.

In the study (SOBOYEJO et al. 1998), a new approach to the crack growth description was suggested. Namely, a multi-parameter equation in the following form was proposed:

$$\frac{da}{dN} = \overline{\alpha_0} \prod_{i=1}^{k} \left(\frac{X_i}{X_{i,o}} \right)^{\alpha_i} = \alpha_o \prod_{i=1}^{k} X_i^{\alpha_i} \tag{18}$$

where $\overline{\alpha_0}$ is a constant of the dimension da/dN, α_i are exponents characteristic of individual variables X_i, and $X_{i,o}$ are reference constants for these variables, and k is the number of input parameters.

Variables X_i may include: the range of the stress intensity factor, stress ratio, crack opening stress, thickness of a specimen or other material, load or environmental parameter. This extension of the Paris relationship is used by engineers for a commonly acknowledged crack propagation rate zone, dependent upon numerous factors.

Equation (18), after the application of a logarithm, receives the following form:

$$\ln\left(\frac{da}{dN}\right) = \ln(\alpha_o) + \sum_{i=1}^{k} \alpha_i \ln(X_i) \tag{19}$$

It allows to apply the least squares method with the use of empirical data. This form of equation, which fits specific cases, enables quite a simple determination of durability, using variable separation methods during integration, as well as a survival function (probability of the absence of damage) and a failure function (probability of damage).

In the study (GHONEM, ZENG 1991) the concept of the model of a constant probability of crack growth presented by GHONEM and DORE (1987) was developed. According to that model, the crack front consists of a large number of randomly selected points that can propagate in any direction under the influence of cyclic loads. The crack surface is divided into equally distant states and the width of each state equals the expected error Δx. Treating the crack growth process as remaining, with the probability P_r, in discrete states r after the applied number of cycles N, where:

$$\ln P_r(N) = -\int \lambda_r dN + L \tag{20}$$

the authors suggest that the parameter λ_r of the intensity of transition into the state r has the form similar to that proposed in (DITLEVSEN, SOBCZYK 1989):

$$\lambda_r = C_1 \cdot f_1(\Delta\sigma, R) \cdot f_2(a) \cdot (\Delta N)^n = C_1 \cdot f_3(\Delta K_{eff}) \cdot (\Delta N)^n \tag{21}$$

With the initial condition $P_r(N) = 1$ for $N = 0$, the following formula is obtained:

$$\Delta N = f_3(\Delta K_{eff})(-\ln P_r(N))^{\beta} \tag{22}$$

where L and β describe the initial state of a crack, and ΔK_{eff} is an effective range of the stress intensity factor.

This formula gives the number of cycles that are indispensable for the crack tip to grow from the state r to the state r+1, i.e. from the crack length $r\Delta x$ to the crack length $(r+1)\Delta x$ with the probability $P_r(N)$. Assuming a constant probability $P_r(N)$ for the given growths Δx substituted into the proper relationship ΔK_{eff}, we can talk about the crack growth constant probability curves. In the paper discussed (supported by the *U.S. Air Force Office for Scientific Research*), the test results of CT specimens made of Ti-6Al-4V titanium alloy and the relationship K_{eff} for the load cases with single overloads are presented.

All the above described issues give a general picture of the fatigue crack propagation and point to the problems connected with that effect and the methods of solving them. Indirectly, the problems also indicate what qualities an optimal mathematical model should have to describe correctly the fatigue crack growth. Certainly, it has not been possible so far to create a single universal model which would include all of the effects observed.

A model description of fatigue crack growth requires an estimation of the values of the propagation equation parameters, e.g. C and m in Paris formula. This can be done on the basis of experimental results (N_i, a_i) – the number of crack cycles and crack length, in accordance with, e.g. the numerical procedure (BASTENAIRE et al. 1981, VIRKLER et al. 1978) presented below.

Since the crack length growth Δa that appeared after ΔN cycles can be expressed with the following relationship:

$$\Delta a = C\Delta K^m \Delta N \tag{23}$$

than under conditions of a constant range of the stress intensity factor ΔK, the expected value Δa for small increments is:

$$E(da/\Delta K^m)=CdN \tag{24}$$

By integrating the two sides, the following equation is obtained:

$$E\left[\int_{a_i}^{a_{i+1}} \Delta K^{-m} da\right] = C\left(N_{i+1} - N_i\right) \tag{25}$$

The expected value, described by the right side of equation (24), differs from the real value of the integration expression, and the difference between them can be expressed with the use of the error parameter e_i, with the following notation:

$$\int_{a_i}^{a_{i+1}} \Delta K^{-m} da = C\left(N_{i+1} - N_i\right)\varepsilon_i \tag{26}$$

On the basis of the results of the analyses and suggestions presented by (BASTENAIRE et al. 1981, VIRKLER et al. 1978) the random variable ε_i can be assigned to a log-normal distribution of the median approximately equal to unity and a constant variance (usually the dispersion of test points log(da/dN)-log(ΔK) is observed within the interval of a specified width). After finding the logarithm:

$$\ln \int_{a_i}^{a_{i+1}} \Delta K^{-m} da = \ln(C) + \ln\left(N_{i+1} - N_i\right) + \ln\left(\varepsilon_i\right) \tag{27}$$

on the right-hand side we get a linear relationship for ln(C), whereas on the left-hand side the relationship for m is no longer linear. As long as the crack length growth is small and ΔK stays more or less constant, the whole left side of the equation can be also approximately treated as linear with respect to m. In general, however, such a linearity cannot be assumed, but, e.g. in the methods presented in (VIRKLER et al. 1978), the value of the parameter m can be determined applying the numerical integration method, assuming a gradual growth of m until the final value is reached with satisfactory accuracy. This allows to perform calculations within a wider range of the crack length growth a_i, because an approximate linear relationship:

$$\ln \int_{a_i}^{a_{i+1}} \Delta K^{-m} da = f_i(m) \tag{28}$$

ensures solution convergence within a few computational steps.

By developing the right side of the equation into a series (28):

$$\ln \int_{a_i}^{a_{i+1}} \Delta K^{-m} da = f_i(m_0) + (m - m_0)\left[\frac{\partial f_i(m)}{\partial m}\right]_{m_0} \tag{29}$$

the following relationship is obtained:

$$f_i(m_0) - \ln(N_{i+1} - N_i) = \ln(C) - (m - m_0)\left[\frac{\partial f_i(m)}{\partial m}\right]_{m_0} + \ln(\varepsilon_i) \tag{30}$$

Since the following also proceeds:

$$\left[\frac{\partial f_i(m)}{\partial m}\right] = \left(\frac{\partial}{\partial m}\int_{a_i}^{a_{i+1}} \exp[-m\ln(\Delta K)]da\right) \Bigg/ \left(\int_{a_i}^{a_{i+1}} \exp[-m\ln(\Delta K)]da\right) =$$

$$= -\left(\int_{a_i}^{a_{i+1}} \ln(\Delta K)\exp[-m\ln(\Delta K)]da\right) \Bigg/ \left(\int_{a_i}^{a_{i+1}} \exp[-m\ln(\Delta K)]da\right) \tag{31}$$

therefore also $f_i(m_0)$ and $\left[\frac{\partial f_i(m)}{\partial m}\right]_{m_0}$ can be calculated with the use of numerical integration methods. As a result, the dependence expressing the expected value of the coefficient m has the form:

$$y_i = \alpha + \beta x_i + \eta_i \tag{32}$$

where:

$x_i = -\left[\frac{\partial f_i(m)}{\partial m}\right]_{m_0}$,

$y_i = f_i(m_0) - \ln(N_{i+1} - N_i)$,

$\alpha = \ln(C)$,

$\beta = m - m_0$,

$\eta_i = \ln(\varepsilon_i)$,

for which, using the least squares method, we can determine the estimators a and b of the parameters α and β and successive approximations of the parameter m, starting with m_0. In successive iterative steps, assuming that $m_1 = m_0 + b$, $m_2 = m_1 + b$, etc., it is possible to determine the expected value of the parameter m the moment convergence is reached.

This method of calculation can be used in all cases of test result analysis in the materials that fulfill the relationship of the Paris equation type. This comprises a large group of issues because, as a rule, the relationship da/dN as a function of ΔK is mostly linear (except for the threshold and critical stress intensity factor zone). This certainly requires an operational knowledge of numerical integration methods as well as regression methods with the use of an experimental data set. Nevertheless, it seems to be a very effective way of propagation equation parameters estimation. Unfortunately, a major disadvantage in this case is that there is no possibility to use this algorithm for variable amplitude load cases, including random loads. The problems connected with overloads and underloads that occur in load spectra (especially operational load spectra), delay and acceleration of the fatigue crack propagation rate and, in consequence, with fatigue life estimation under different load conditions together with the evaluation of the load sequence effect on fatigue life, cannot be modeled with the help of this mathematical apparatus. It seems that in such cases the only solution is to apply elaborate calculation models of fatigue crack growth – taking into consideration the above effects – characterized by stochastic or determined parameters referring to, e.g. material constants and fatigue properties, load spectra and other elements of fracture mechanics.

From the perspective of construction element life estimation, the knowledge of the elementary material fatigue characteristics is important not only at the design stage but also during its service period and while analyzing the service life evaluation and the possibilities of its prolonging. This especially refers to the fatigue crack propagation characteristics within a wide load range, their relationships, e.g. with the stress intensity factor, the stress ratio. In the case of, e.g. aviation structures, it is directly connected with flight safety (when the cracks reach a critical length, the construction element is damaged). In consequence, the economic part of aviation is affected - costs of failures and disasters are huge. However, taking out of service the elements that still have a fatigue life margin is also a loss. Yet, the loss can be minimized if the recognition and control of a damaging process are more efficient. Gaining the knowledge about, e.g. fatigue crack growth under load conditions that are as close to the operational conditions as possible, is indispensable for durability analyses.

Below are presented the results of fatigue crack propagation tests and analyses. The analyses were conducted using mathematical apparatus and elements of fracture mechanics. Their aim was to determine a difference stemming from the selection of specimen types for the tests. In order to reach a satisfactory estimation of the fatigue crack growth process under operational conditions, the specimens that closely represent real conditions should be applied. In general, fatigue crack growth tests are carried out with the use of normalized specimens since this ensures the possibility of result comparison for different kinds of materials and loads as well as the application of a homogeneous mathematical apparatus and testing equipment. Specimens used for fatigue tests can be classified into two groups, depending on how the load is applied (SCHIJVE 1998):

– symmetric (Middle Crack Tension $M(T)$ or Center Crack Tension $CC(T)$, Central Notch Tension $CN(T)$, Double Edge Notch Tension $DEN(T)$, Two-hole Tension $TH(T)$),
– asymmetric (Compact Tension $C(T)$, Round Compact Tension $RC(T)$, Extended Compact Tension $EC(T)$, Circular Edge Notch Tension $CEN(T)$, Single Edge Notch Tension $SEN(T)$).

These specimens can be stretched either by a bolt inserted in the holes that are in them or by holding a part of a specimen with machine grips. In the case of asymmetrically loaded specimens, they are stretched and bent at the same time, and together with an increment in the crack length, the bending moment increases with respect to their non-cracked part. The stress intensity factor increases along with an increase in the crack length a and with a decrease in the geometric dimensions B, W. It happens faster than in the case of symmetric specimens. Fig. 1 shows a change in the value of the shape function Y, occurring during the calculation of the stress intensity factor ΔK (33), depending on the dimensionless crack length a/W, according to the form given in references (SCHIJVE 1998, *Stress Intensity Factors Handbook* 1987):

for specimens $C(T)$, $RC(T)$, $EC(T)$ – in the notation including the load force F:

$$\Delta K = \frac{\Delta F}{B\sqrt{W}} \cdot Y \tag{33}$$

for specimens $CC(T)$, $DEN(T)$, $SEN(T)$ – in the notation including the nominal stress σ:

$$\Delta K = \Delta\sigma\sqrt{\pi a} \cdot Y \tag{34}$$

There is a significant difference between the curves of symmetric and asymmetric specimens. In the case of the latter – the change is minor almost over the entire range of crack length and to such a degree it affects an increase in the stress intensity factor and the crack length growth. Below there is a comparison of the fatigue crack growth rates in 18G2A steel under the same load conditions, in selected specimens from both groups: with an edge notch – specimen type $SEN(T)$, and with a center crack – specimen type $CC(T)$, (KŁYSZ 2003).

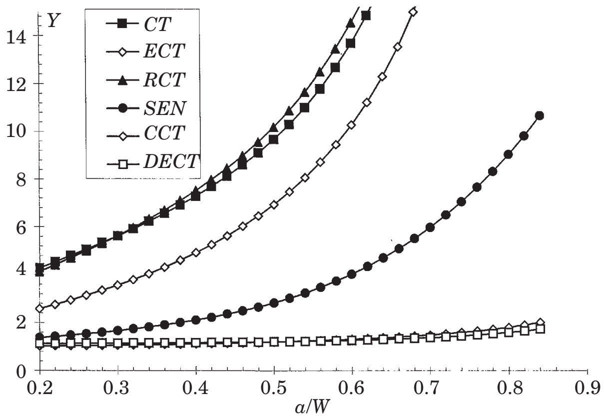

Fig. 1. Relationship of the shape function Y of the stress intensity factor and the dimensionless crack length a/W for symmetric and asymmetric specimens

The shape of the specimens tested in the study is shown in Fig. 2. The specimens had identical geometric dimensions (width, length, thickness) and were made of the same batch of material – the only difference is the above-mentioned notch location. The dimensions of the specimens were the following: length $L = 245$ mm, width $W = 60$ mm, thickness $B = 8$ mm, initial crack length $a_o = 6$ mm. Test conditions: constant load amplitude in base

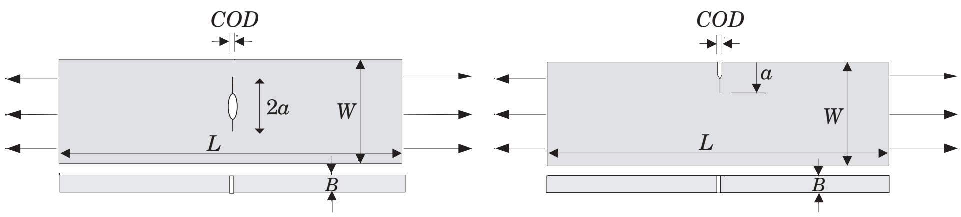

Fig. 2. Shape and dimensions of $CC(T)$ and $SEN(T)$ specimens

cycles F_{min}– F_{max}, stress ratio $R = 0.3$, values of overloads k_{ov} and the distances between them DN – as in Table 1. The same method of crack length determination was used, i.e. the compliance method, with the use of a crack opening displacement detector of the clip gauge type – Fig. 3 (KŁYSZ 2003).

The stress intensity factor for standard specimens has the following form (LIU 1974, *Stress Intensity Factors Handbook* 1987):

– for specimen $SEN(T)$:

$$\Delta K = \frac{\Delta F}{B\sqrt{W}}\left(1.12 - 0.231\frac{a}{W} + 10.55\left(\frac{a}{W}\right)^2 - 21.72\left(\frac{a}{W}\right)^3 + 30.39\left(\frac{a}{W}\right)^4\right)$$

or (35)

Table 1

Conditions and results of fatigue tests for two types of specimens

Specyfication	Specimens *SEN*(*T*)				Specimens *CC*(*T*)			
F_{min}– F_{max} (N)	25 500 – 85 000							
k_{ov}	-	1.4	1.6	1.75	-	1.4	1.4	1.6
DN (cycles)	-	5000	12500	15000	-	5000	12500	12500
a_0 (mm)	8.95	9.07	8.88	8.91	9.25	9.90	10.61	9.90
$A_{critical}$ (mm)	38.03	34.20	26.59	29.29	13.70	16.15	21.14	16.46
$\Delta K_{initial}$ (MPa$\sqrt{m}$)	39.1	39.3	39.1	39.1	22.9	23.5	24.8	24.0
$\Delta K_{critical}$ (MPa$\sqrt{m}$)	141.0	112.1	73.3	84.1	30.2	34.3	46.1	35.6
$2N_f$ (cycles)	72 559	95 000*	125 000*	195 000*	59 611	50 000*	52 500*	62 500*

* The specimen cracked in an overloading cycle

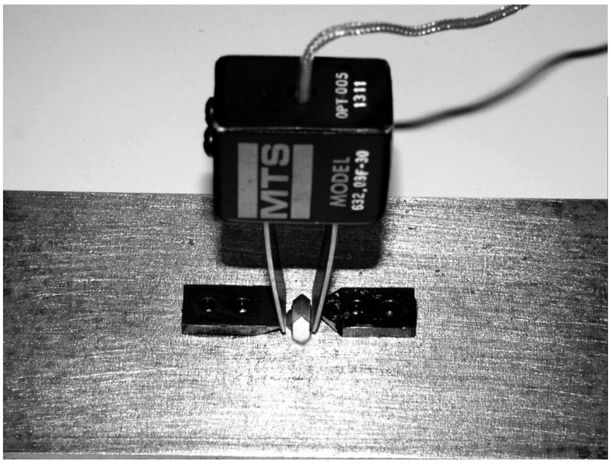

Fig. 3. Specimen type *CC*(*T*) with a crack opening displacement detector of a clip gauge type

$$\Delta K = \frac{\Delta F}{B\sqrt{W}} \frac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{1.5}}$$

$$\left(0.534-0.6454\frac{a}{W}+1.29125\left(\frac{a}{W}\right)^2-1.52958\left(\frac{a}{W}\right)^3+0.8998\left(\frac{a}{W}\right)^4\right)$$

– for specimen $CC(T)$:

$$\Delta K = \frac{\Delta F}{B\sqrt{W}}\sqrt{\frac{\pi a}{W}\sec\left(\frac{\pi a}{2W}\right)}$$

or (36)

$$\Delta K = \frac{\Delta F}{B\sqrt{W}} \frac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{1.5}}$$

$$\left(0.5002-1.0015\frac{a}{W}+0.7873\left(\frac{a}{W}\right)^2-0.4661\left(\frac{a}{W}\right)^3+0.1771\left(\frac{a}{W}\right)^4\right)$$

and a compliance function takes the form:

– for specimen $SEN(T)$:

$$\frac{a}{W}=1-4.0632u+11.242u^2-106.04u^3+464.33u^4-1650.68u^5$$

or (37)

$$\frac{a}{W}=-1.9797+37.625u-198.384u^2+480.643u^3-565.123u^4+261.128u^5$$

– for specimen $CC(T)$:

$$\frac{a}{W}=1.069u+0.5881u^2-1.01885u^3+0.36169u^4$$

or (38)

$$\frac{a}{W} = -34.5136 + 100.205u - 114.033u^2 + 64.4959u^3 - 18.0333u^4 + 1.9902u^5$$

where: compliance $u = \dfrac{1}{1 + \left(\dfrac{E \cdot B \cdot COD}{\Delta F} \right)^{0.5}}$,

a/W – dimensionless crack length,
B – specimen thickness,
ΔF – load range F_{max}–F_{min},
E – Young's modulus,
COD – Crack Opening Displacement.

Test results in the form of crack length relationship as a function of the number of cycles $a = f(N)$ and crack growth rate as a function of the stress intensity factor range $da/dN = f(\Delta K)$ are presented in Fig. 4.

There is a distinct difference between the courses of these two relationships, not only between the individual specimens *SEN*(*T*) and *CC*(*T*) but also between their types. The effects of the loads applied are clearly visible in all curves, however, the character of their changes for the specimens of diffe-

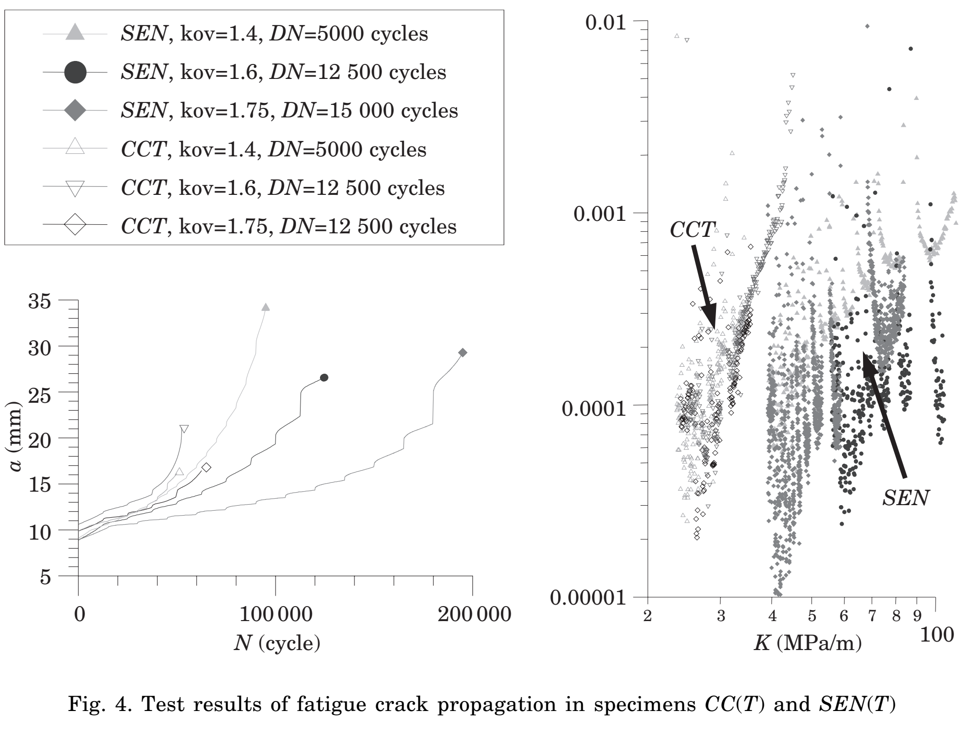

Fig. 4. Test results of fatigue crack propagation in specimens *CC*(*T*) and *SEN*(*T*)

rent type is contrasting. Therefore, the fatigue crack growth parameters in specimens $SEN(T)$ and $CC(T)$ to which the same load conditions were applied, differ as for crack lengths, stress intensity factors and crack propagation rates. The situation does not change even when the crack length in the form of a, a/W, $2a$ or $2a/W$ in the relationships $a = f(N)$ and $da/dN = f(\Delta K,)$ in case of $CC(T)$ specimens, is taken into consideration. In the case of a specimen with a central notch, the crack grows simultaneously in two directions, which seems to be a decisive factor (as a result there is a faster increase in the dimensionless length value $2a/W$, compared with the value a/W for specimen $SEN(T)$). With the same degree of external load, the flexibility of specimen $CC(T)$ changes faster. Under corresponding load conditions, the test results of crack growth in different specimens are diverse and thus cannot be compared unselectively. Hence, the selection of a specimen type is important for the fatigue tests simulating operational conditions, and the application of the equation coefficients describing fatigue crack growth that can be found in professional literature should be preceded by a detailed analysis of their compatibility with respect to the tests conducted.

The specimens discussed above provide various kinds of crack growth conditions. In consequence, this makes the representation of different cases of real construction elements and operational conditions possible. One of the selection criteria of the specimen types used in tests should be a guarantee of such crack growth conditions that are as similar to the real ones as possible – especially when it comes to crack geometry, type and method of load transfer. For example, in aviation construction elements, like a compressor disk or blade, made of titan alloy or a high-strength steel of the maraging type, there are no cracks that start growing from holes of technological notches or that grow under symmetric load conditions. Accordingly, it might be assumed that in the analysis of fatigue crack growth problems, it will be more correct if the parameter determination of this growth for these elements is carried out with on edge crack specimens, e.g. Compact Tension and Round Compact Tension. These specimens, when compared with central crack specimens or notch specimens, are a more accurate representation of real failures, scratches or defects due to foreign matters that appear during service.

A correct representation based on the selection of test specimens, load conditions similar to those that occur during service (symmetric or asymmetric, or in a plane stress or strain state) and used in the analyses of construction element crack growth is fundamental while deciding whether the determined characteristics can be treated as the closest to the real ones.

Summary

The study presents an analysis of problems related to the estimation of the fatigue life of elements and constructions. It discusses the currently developed fatigue test methods as well as procedural algorithms applied in the above analysis. It was demonstrated that these problems are interdisciplinary and thus require a cooperation of specialists representing many different branches of science.

In general, there is a lack of homogenous, universal and commonly applied solutions concerning e.g. the shape function of the stress intensity factor, used in the analyses of construction elements characterized by a complex shape and load conditions. In relation to such elements, certainly the most reliable are the tests on the 1:1 scale, those carried out on real objects and combined with the tests of a whole construction for the best results. However, this requires research facilities, measuring equipment and economic resources. On the grounds of the results of analyses, we can estimate the effects of different factors on the service process of elements or constructions. The knowledge of real service load of construction elements is indispensable and crucial for a correct evaluation of structure safety, the loss or margin of its life.

The analyses presented in this paper are an example of the application of experimental data, numerical calculations FEM characteristic of the element tested, and its load conditions, and fatigue life estimation mathematical models that result in quantitative (initial) and qualitative (more reliable) evaluations of the durability of a given construction element. Each of the components of this analysis makes a significant and mutually complementary contribution to a reliable fatigue life estimation. Only the combination of various branches of science creates great possibilities as for practical analyses of construction element damaging mechanisms. Having these kind of effective analytical tools at our disposal, it is possible to rationally minimize empirical testing of expensive equipment under real conditions, and properly plan laboratory tests. They should generally deal with material characteristics and load spectrum estimation of individual construction elements, as well as with the simulation of environmental conditions specific to a given service process. As regards analytical methods of durability estimation, they should, with time, become more and more precise, and the models should represent real relationships and properties of the objects tested more faithfully.

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