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EFFECTIVE LENGTHS OF REINFORCED CONCRETE COLUMNS IN SINGLE-STOREY FRAME STRUCTURES IN THE LIGHT OF THE EUROCODE

***Michał Knauff*[1], *Krzysztof Klempka*[2]**

[1] Institute of Building Structures
Warsaw University of Technology
[2] Department of Civil Engineering and Building Structures
University of Warmia and Mazury in Olsztyn

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A b s t r a c t

In the following article we present the application of the Eurocode rules for calculating the effective length of columns in single-storey buildings. Assuming the same effective lengths for cantilevers (i.e. $l_0 = 2l_{col}$) and the construction columns, as in the example, will not always be correct. The problem of calculating the effective length mostly presents itself when using the simplified method. In the exact method it is not required to determine the effective lengths. This is why it is advisable while designing to use the exact method based on second order analysis and taking into account the nominal stiffness.

DŁUGOŚCI OBLICZENIOWE SŁUPÓW ŻELBETOWYCH W PARTEROWYCH UKŁADACH RAMOWYCH W ŚWIETLE PRZEPISÓW EUROKODU

***Michał Knauff*[1], *Krzysztof Klempka*[2]**

[1] Instytut Konstrukcji Budowlanych
Politechnika Warszawska
[2] Katedra Budownictwa i Konstrukcji Budowlanych
Uniwersytet Warmińsko-Mazurski w Olsztynie

S ł o w a k l u c z o w e: długość obliczeniowa słupa, metoda nominalnej sztywności, efekty drugiego rzędu, hale żelbetowe, konstrukcje żelbetowe.

A b s t r a k t

Rozpatrzono zastosowanie zasad Eurokodu do wyznaczania długości obliczeniowych słupów w jednokondygnacyjnych budynkach halowych. Przyjmowanie długości obliczeniowych l_0 jak dla

wsporników (tzn. $l_0 = 2l_{col}$) dla słupów w takich halach, jak w przykładzie, nie zawsze będzie prawidłowe. Problem wyznaczania długości obliczeniowej dotyczy przede wszystkim metody uproszczonej. Zastosowanie metody ścisłej nie wymaga określania długości obliczeniowych, dlatego do projektowania zaleca się stosowanie metody ścisłej polegającej na analizie II rzędu z uwzględnieniem sztywności nominalnych.

Introduction

In 2010, Eurocode EN 1992-1-1 (abbr: EN) will replace the Polish norm PN-B-03264 (abbr: PN) for designing reinforced concrete and pre-stressed constructions, which will in turn lead to many other changes in the design process. The EN calculating methods differ in many ways from the PN methods, and one of the differences are the principles for calculating the effective lengths of columns. The EN contains extensive rules referring to the effective length of isolated elements, but it does not contain any rules corresponding to the Polish rules for calculating single-storey structures according to the Appendix C of the PN. According to the PN, it can be assumed that $l_0 = 1{,}6l_{col}$ (when the roof construction is rigid), while according to the older versions of the PN even $l_0 = 1{,}2l_{col}$ “when there are four or more columns”. After implementing the EN the recommendations regarding the effective lengths enclosed in the Appendix C of the PN will no longer act as rules of the norm. While calculating columns for single-storey buildings, as in the example, according to the EN it is required to assume that the effective length is the same as for the cantilevers (i.e. $l_0 = 2l_{col}$) – the EN has no rules which would allow engineers to assume any other effective length. We are thus faced with a question whether the assumptions of the EN referring to the effective lengths are correct. To answer this question we compared the values of bending moments for columns in a single-storey building obtained according to the method based on nominal stiffness (a simplified method in the EN) with the values obtained according to the exact method based on the second order analysis and taking into account the nominal stiffness. Later on in the article we present a short description of the method based on nominal stiffness and a derivation of the formula for a coefficient increasing the moment. We finish with examples of calculating a single-storey building.

Method based on nominal stiffness

This method is based on the fact that in the second order analysis are applied constant (i.e. load independent) values of stiffness, also called nominal stiffness, obtained from simple approximations of the flexural stiffness, smaller than the initial stiffness, calculated taking into account the influence of cracking, non-linear qualities of materials and creep. In order to apply this method to the whole framed structure it is necessary to carry out static calculations according to the first order theory taking into account the nominal stiffness of columns and then to increase the derived bending moments M_{0Ed} according to the following formula:

$$M_{Ed} = \eta M_{0Ed} \qquad \eta = \left(1 + \frac{\beta}{\frac{N_B}{N_{Ed}} - 1}\right) \tag{1}$$

where

$$\beta = \frac{\pi^2}{c_0},$$

c_0 is a coefficient which depends on the distribution of first order moment (for instance, $c_0 = 8$ for a constant first order moment, $c_0 = 9{,}6$ for a parabolic and 12 for a symmetric triangular distribution etc.).

Formula (1) takes into account the second order effects and was derived for a column like in Figure 1 loaded with the longitudinal force N_{Ed} and any transverse load. w_0 is a deflection calculated according to the first order theory and w is a total deflection of column. The deflection half way through the length of the element $w_{\max}$ is a sum of the deflection according to the first order theory $w_{0\max}$ and the increment Δw caused by the moment of force N_{Ed} towards the deformed axis of the element. It was assumed that the deflection can be approximately calculated from the formula

$$w = w_{\max} \sin \frac{\pi x}{l_o}$$

After applying the Maxwell-Mohr formula we obtained:

$$\Delta w = \frac{N_{Ed}\, w_{\max} l_o^2}{\pi^2 EI} = N_{Ed} \frac{w_{\max}}{N_B}$$

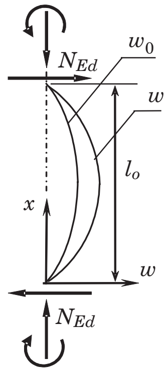

Fig. 1. A diagram of a column

In the above formula *EI* is the flexural rigidity of a compressed element (nominal stiffness), and N_B is a buckling load obtained from the Euler's formula

$$N_B = \frac{\pi^2 EI}{l_o^2} \tag{2}$$

Total deflection

$$w_{\max} = w_{0\max} + \Delta w = w_{0\max} + \frac{N_{Ed}}{N_B} w_{\max}$$

thus

$$w_{\max} = w_{0\max} \frac{1}{1 - \dfrac{N_{Ed}}{N_B}} \tag{3}$$

Half way through the length of the element the bending moment according to the second order theory

$$M_{Ed} = M_{0Ed} + N_{Ed} w_{\max} \tag{4}$$

Substituting in (3) $w_{\max}$ derived from (4) we obtain

$$\frac{M_{Ed} - M_{0Ed}}{N_{Ed}} = w_{0\max} \frac{N_B}{N_B - N_{Ed}},$$

$$M_{Ed} = M_{0Ed} + w_{0\max} \frac{N_B}{\dfrac{N_B}{N_{Ed}} - 1} = M_{0Ed} \left(1 + \frac{\dfrac{w_{0\max} N_B}{M_{0Ed}}}{\dfrac{N_B}{N_{Ed}} - 1} \right) \tag{5}$$

The deflection $w_{0\max}$ may be expressed by the following formula

$$w_{0\max} = \frac{M_{0\max} l_o^2}{c_0 B} \tag{6}$$

Placing (6) in (5) we obtain formula (1) recommended by the norm. The derivation of formula (1) was also presented in other articles, e.g. KLEMPKA, KNAUFF (2005).

The effective length of a column according to the EN may be derived by transfroming formula (2) to the following form

$$l_0 = \pi \sqrt{\frac{EI}{N_B}} \tag{7}$$

Then substituting N_B derived from (3) in formula (7) we obtain:

$$l_0 = \pi \sqrt{\frac{EI}{N_{Ed}} \left(1 - \frac{w_{0\max}}{w_{\max}} \right)}$$

Using the above formula we can derive the buckling coefficient

$$\mu = \frac{\pi}{l_0} \sqrt{\frac{EI}{N_{Ed}} \left(1 - \frac{w_{0\max}}{w_{\max}} \right)} \tag{8}$$

The examples of calculations

In the following examples the values of bending moments were calculated in columns of two-nave buildings using the method based on nominal stiffness and the exact method. We assumed that the horizontal force caused by wind pressure and suction equals $H = 30$ kN. We also assumed that the rigid construction of the roof forces equal horizontal shifts of the top ends of columns. The columns have identical cross-sections $b = 40$ cm, $h = 45$ cm, concrete C40/50, and steel A-III. The edge columns' reinforcement is 4 ϕ 16 ($A_s = 8{,}04$ cm^2), and the internal columns' reinforcement is 7 ϕ 16 ($A_s = 14{,}07$ cm^2) on each side of the cross-section, $a = 3{,}5$ cm.

The calculations were carried out for two different longitudinal loads:

Case 1

$P_1 = 200$ kN in edge columns,

$P_2 = 900$ kN in internal column.

Case 2

$P_1 = 450$ kN in edge columns,

$P_2 = 790$ kN in internal column.

The calculations for case 1 are presented below.

Imperfections according to point 5.2 of the EN

$$\alpha_h = \frac{2}{\sqrt{l}} = \frac{2}{\sqrt{7{,}0}} = 0{,}756,\ \frac{2}{3} \le 0{,}756 \le 1{,}0,\ \alpha_m = \sqrt{0{,}5(1+1/\mathrm{m})} =$$

$$= \sqrt{0{,}5(1+1/3)} = 0{,}816,$$

$$\theta_h = \theta_0\, \alpha_h\, \alpha_m = \frac{1}{200}\, 0{,}756 \cdot 0{,}816 = 0{,}00308.$$

Horizontal forces caused by imerfections:

– In edge columns $H_1 = \theta_i\, P_1 = 0{,}00308 \cdot 200 = 0{,}616$ kN,

– In internal column $H_2 = \theta_i\, P_2 = 0{,}00308 \cdot 900 = 2{,}772$ kN.

Calculations according to the EN – the method based on nominal stiffness

Moments according to the first order theory caused by forces H_i, assuming that the stiffness of columns is identical, are presented in the diagram marked as $\mathrm{ENI_{const}}$ (Fig. 2).

The nominal stiffness (point 5.8.7.2 of the EN) depends on the amount of reinforcement. While designing columns the reinforcement should always be taken into account because it allows us to calculate the increased bending moments, and based on these moments we can calculate the necessary reinforcement. The result is obtained through iteration after reaching a reasonable conformity between the assumed and calculated reinforcement.

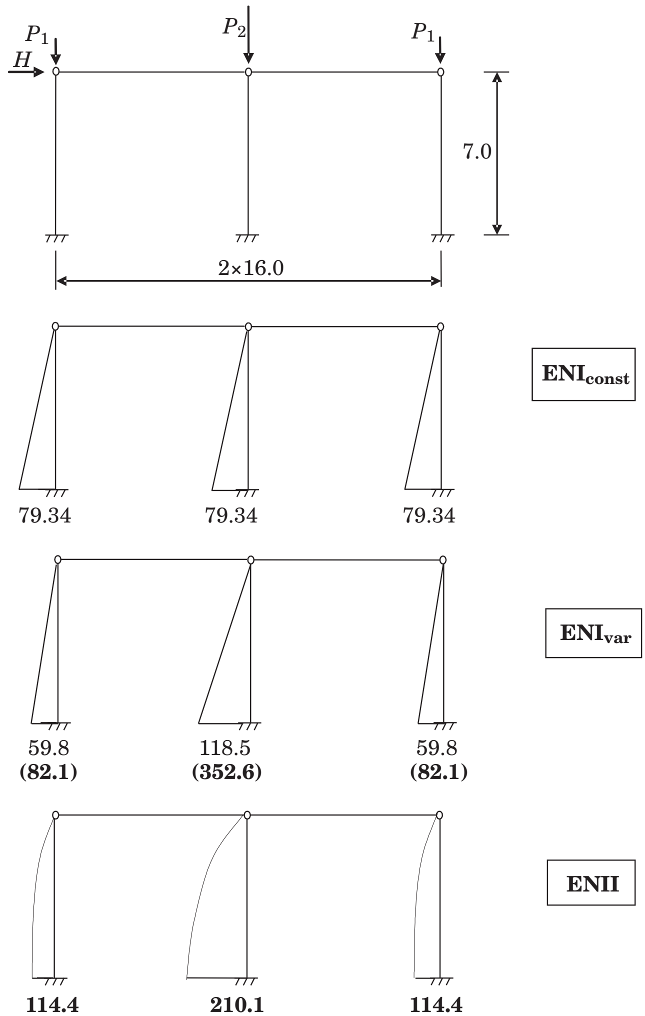

Fig.2. Static diagram and bending moments in columns [kNm]- case 1, the values in brackets were determined using formula (1), the abbreviations ENI_{const}, ENI_{var}, ENII are explained in the text

The design value of modulus of elasticity of concrete E_{cd} = 29170 MPa, the moment of inertia $I_c = 3{,}038 \times 10^{-3}$ m^4. Coefficients k_1 and k_2 according to point 5.8.7.2 of the EN:

$k_1 = \sqrt{f_{ck}/20} = \sqrt{40/20} = 1{,}414$. For $l_0 = 2l_{col} = 2 \cdot 7 = 14{,}0$ m, the radius of inertia $i = \dfrac{h}{2\sqrt{3}} = \dfrac{0{,}45}{2\sqrt{3}} = 0{,}1299$ m. Slenderness $\lambda = l_0 \,/\, i = 14{,}0 \,/\, 0{,}1299 = 107{,}8$. We assumed the effective creep ratio $\varphi_{ef} = 1{,}945$. The coefficient k_2 for the edge columns

$$n = N_{Ed} / (A_c f_{cd}) = 200/(0{,}40 \times 0{,}45 \times 26{,}7 \times 10^3) = 0{,}0416,$$

$$k_2 = n \cdot \frac{\lambda}{170} = 0{,}0416 \, \frac{107.77}{170} = 0{,}0264 \leq 0{,}20.$$

The moment of inertia of the reinforcement

$$I_s = 2A_s(h/2 - a_1)^2 = 2 \times 8{,}04 \times 10^{-4}(0{,}45/2 - 0{,}035)^2 = 5{,}8 \times 10^{-5} \text{ m}^4.$$

Coefficient $K_c = k_1 k_2 / (1 + \phi_{ef}) = 1{,}414 \times 0{,}0264 / (1 + 1{,}945) = 0{,}0127$ i $K_s = 1$

Nominal stiffness of the edge columns:

$$EI = K_c E_{cd} I_c + K_s E_s I_s = 0{,}0127 \times 29170 \times 10^3 \times 3{,}0375 \times 10^{-3} + 1{,}0 \times$$
$$\times 200 \times 10^6 \times 5{,}8 \times 10^{-5} = 12{,}73 \text{ MNm}^2.$$

In the internal column

$$n = N_{\mathrm{Ed}} / (A_c f_{cd}) = 900/(0{,}40 \times 0{,}45 \times 26{,}7 \times 103) = 0{,}1873,$$

$$k_2 = n \cdot \frac{\lambda}{170} = 0{,}1873 \, \frac{107.77}{170} = 0{,}118 \leq 0{,}20,$$

$K_c = k_1 k_2 / (1 + \varphi_{ef}) = 1{,}414 \times 0{,}118 / (1 + 1{,}945) = 0{,}0566$ i $K_s = 1$.

Moment of inertia of the reinforcement

$$I_s = 2A_s(h/2 - a_1)^2 = 2 \times 14{,}07 \times 10^{-4}(0{,}45/2 - 0{,}035)^2 = 10{,}15 \times 10^{-5} \text{ m}^4.$$

Nominal stifffness of the internal column:

$$EI = K_c E_{cd} I_c + K_s E_s I_s = 0{,}0566 \times 29170 \times 10^3 \times 3{,}0375 \times 10^{-3} + 1{,}0 \times$$
$$\times 200 \times 10^6 \times 10{,}15 \times 10^5 = 25{,}31 \text{ MNm}^2.$$

The result of calculations carried out according to the first order theory with nominal stiffness of columns is presented in Figure 2 – the diagram marked as $\mathrm{ENI}_{\mathrm{var}}$.

Incresed bending moments:

a) In the edge column:

$$\text{Buckling load } N_B = \frac{\pi^2}{l_0^2} EI = \frac{\pi^2}{14^2} 12{,}73 = 0{,}64077 \text{ MN},$$

$$M_{Ed} = M_{0Ed} \left(1 + \frac{\frac{\pi^2}{12}}{\frac{N_B}{N_{\mathrm{Ed}}} - 1} \right) = 59{,}8 \left(1 + \frac{0{,}8225}{\frac{640{,}77}{200} - 1} \right) = 82{,}1 \text{ kNm}.$$

b) In the internal column:

$$\text{Buckling load } N_B = \frac{\pi^2}{l_0^2} EI = \frac{\pi^2}{14^2} 25{,}31 = 1{,}27471 \text{ MN},$$

$$M_{Ed} = M_{0Ed} \left(1 + \frac{\frac{\pi^2}{12}}{\frac{N_B}{N_{Ed}} - 1} \right) = 118{,}515 \left(1 + \frac{0{,}8225}{\frac{1274{,}71}{900} - 1} \right) = 352{,}6 \text{ kNm}.$$

Calculations according to the EN – second order theory, nominal stiffness

Using the stiffness derived in the previous point we carry out static calculations according to the second order theory – the result diagram marked as ENII is presented in Figure 2.

Additionally, the buckling coefficients were calculated using formula (8) for the edge columns

$$\mu = \frac{\pi}{l_0} \sqrt{\frac{EI}{N_{Ed}} \left(1 - \frac{w_{0\max}}{w_{\max}} \right)} = \frac{\pi}{7} \sqrt{\frac{12725{,}14}{200} \left(1 - \frac{0{,}0765}{0{,}1546} \right)} = 2{,}5$$

and the internal column

$$\mu = \frac{\pi}{l_0} \sqrt{\frac{EI}{N_{Ed}} \left(1 - \frac{w_{0\max}}{w_{\max}} \right)} = \frac{\pi}{7} \sqrt{\frac{25314{,}41}{900} \left(1 - \frac{0{,}0765}{0{,}1546} \right)} = 1{,}69$$

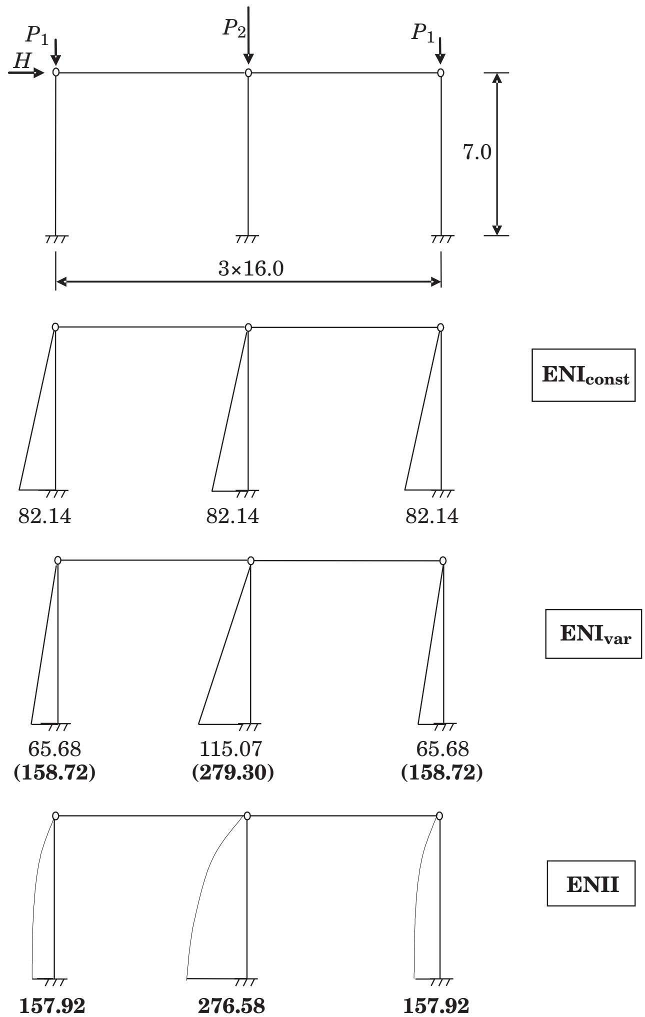

Fig. 3. A static diagram and bending moments in columns [kNm]- case 2, the values in brackets were determined using formula (1), the abbreviations ENI_{const}, ENI_{var}, ENII are explained in the text

Values of w_{0max} and w_{max} were obtained using the exact method.
The result of calculations for case 2 are presented in Figure 3.
The buckling coefficients for the edge columns:

$$\mu = \frac{\pi}{l_0}\sqrt{\frac{EI}{N_{Ed}}\left(1-\frac{w_{0max}}{w_{max}}\right)} = \frac{\pi}{7}\sqrt{\frac{14124,92}{450}\left(1-\frac{0,0759}{0,2047}\right)} = 2,0$$

and the internal column:

$$\mu = \frac{\pi}{l_0}\sqrt{\frac{EI}{N_{Ed}}\left(1 - \frac{w_{0\max}}{w_{\max}}\right)} = \frac{\pi}{7}\sqrt{\frac{24729{,}69}{790}\left(1 - \frac{0{,}0759}{0{,}2047}\right)} = 2{,}0$$

In the second case the reliable moments are similar to the reliable moments calculated using the method of increasing the moment, while in the first case these values are different. This diffeerence results from a false assumption that in case 1 each of the columns in the frame behaves in the same way as an isolated cantilever, which means that identical effective lengths were assumed for all the columns and the cantilevers ($\mu = 2$). Such an assumption can only be made when the ratios of the columns' stiffness *EI* to the longitudinal forces acting in them N_{Ed} are identical, which results from formula (8). This conclusion refers to cases in which the roof construction forces equal horizontal shifts of the top ends of columns, i.e. cases in which the ratio $\frac{w_{0\max}}{w_{\max}}$ is identical for each column.

Conclusions

Considering the presented analyses, we can conclude that:

1. In a single-storey building with columns joined monolithically with the foundations and via hinges with the roof construction the buckling coefficient for each column is 2,0 only when the ratios of the columns' stiffness *EI* to the longitudinal forces acting in them N_{Ed} are identical. It may not always be possible to meet such a condition, so assuming that $\mu = 2{,}0$ for each column will not always be correct.
2. The problem of calculating the effective length mostly refers to the simplified method. While applying the exact method it is not required to define the effective lengths – the shape of deflection and the resulting increase of the moments are calculated directly. This is why while designing it is advisable to use the exact method based on the second order analysis and taking into account the nominal stiffness.

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