TECHNICAL SCIENCES
Abbrev.: Techn. Sc., No 11, Y 2008

DOI 10.2478/v10022-008-0030-z

TRUSS ANALYSIS IN VIEW OF THE REPLACEMENT OF DEGRADED STRUCTURAL COMPONENTS

Waldemar Dudda

Department of Mechanical Engineering and Fundamentals of Machine Design
University of Warmia and Mazury in Olsztyn

K e y w o r d s: numerical analysis, cyclic loading, corrosion degradation, energy-based criterion.

A b s t r a c t

The study involved a numerical analysis of trusses under cyclic loading. Particular components of such structures are exposed to plastic deformation and corrosion damage and they dissipate energy, including the contributions of irreversible phenomena such as the work of non-elastic strains. The lifetime of individual structural components may be estimated applying the energy-based criterion. During numerical simulation, the most degraded component was replaced after the set number of load cycles. The aim of the analysis was to determine the optimal cycle during which a given component should be replaced with a new one. The paper presents the results of calculations regarding a sample truss construction. An analysis of those results provided a basis for determining the optimal cycle during which the most degraded component should be replaced.

ANALIZA KONSTRUKCJI W ASPEKCIE WYMIANY DEGRADUJĄCYCH SIĘ ELEMENTÓW

Waldemar Dudda

Katedra Mechaniki i Podstaw Konstrukcji Maszyn
Uniwersytet Warmińsko-Mazurski w Olsztynie

S ł o w a k l u c z o w e: analiza numeryczna, obciążenie cykliczne, degradacja korozyjna, kryterium energetyczne.

A b s t r a k t

Artykuł dotyczy numerycznej analizy konstrukcji kratowych obciążonych cyklicznie. Poszczególne elementy konstrukcji ulegają plastycznej i korozyjnej degradacji oraz dysypują energię, na którą składają się wkłady od nieodwracalnych zjawisk typu praca na odkształceniach niesprężystych. Trwałość poszczególnych elementów konstrukcji jest oceniana według kryterium energetycznego.

W trakcie symulacji numerycznej po zadanej liczbie cykli obciążenia jest wymieniany najbardziej degradujący się element. Analiza ma na celu określenie optymalnego cyklu, w którym powinna nastąpić wymiana elementu na nowy. W pracy przedstawiono wyniki obliczeń przykładowej konstrukcji kratowej. W wyniku analizy otrzymanych wyników określono optymalny cykl wymiany najbardziej degradującego się elementu.

Introduction

The paper presents an analysis of truss behavior, performed using numerical modeling of load cycles. During cyclic loading of a sample truss construction, its structural components are exposed to plastic deformation and corrosion damage, leading to a decrease in the cross-sectional area of truss members. This results in stress increment, particularly in components which are at the greatest risk of attaining the elastic limit, and in material strengthening. At the moment plastic strains occur in successive load cycles, the material dissipates energy corresponding to the work of non-elastic strains. Each type of material is characterized by a certain value of irreversibly dissipated energy. When this value is exceeded, the material undergoes permanent damage. The level of this energy is referred to as critical energy (JAKUBOWSKI 2000). It follows that "numerical damage" of a given truss member takes place when energy dissipation exceeds the critical level during cyclic loading, while the number of the load cycle during which this level is exceeded determines the critical number of load cycles for a given truss construction. During numerical simulation with the use of the finite element method, after the set number of cycles, the most degraded truss member is "regenerated", i.e. reconstituted to fitness for use by restoring its initial parameters including cross-sectional area, mechanical properties and the zero level of dissipated energy. Simulation is continued to determine the critical number of load cycles and the optimal number of cycles after which a given component should be replaced with a new one. Studies of that kind contribute to the development of a new branch of science known as "combined shakedown" which investigates, among others, the modes of material degradation including stress corrosion, electrochemical corrosion, low-cycle corrosion fatigue and failure (DUDDA, BADUR 2001).

Numerical applications

D-KRAT program was used for numerical simulation. The main part of the program is *Mini-Mod* library which contains Finite Element Method solver procedures and enables a non-linear analysis of structures. The solution of a non-linear system of equations is obtained via an incremental-iteration process. Load increments are determined by parameter λ whose increments $\Delta\lambda$ are selected so as to form a closed cycle of loads (DUDDA, BADUR 2000, 2001). Non-linear load paths are tracked using the numerical technique developed by CHRÓŚCIELEWSKI (1996). Another part of the program is *Mini-Kor* module enabling to estimate the degree of degradation of particular structural components. At present this module contains models describing such phenomena as stress corrosion, electrochemical corrosion, high-temperature corrosion and low-cycle corrosion fatigue. It was assumed that the above three types of corrosion take place on the surface of a structural component, and that they lead to a decrease in the bearing surface of a truss member. The rate $\dot{d}$ of thickness decrement of the external layer of material exposed to corrosion can be described by the following formulas (DUDDA 2005):

– stress corrosion

$$\dot{d}_{\mathrm{SC}} = C_{\mathrm{SC}} \, | \, \sigma - \sigma_{gr} \, |^{n} \, e^{(T-T_0)/B} \tag{1}$$

– electrochemical corrosion and high-temperature corrosion

$$\dot{d}_{\mathrm{HC}} = C_{\mathrm{HC}} \, (T/T_0)^{\kappa} \, | \, \nabla T \, |^{m} \tag{2}$$

– low-cycle corrosion fatigue

$$\dot{d}_{\mathrm{LC}} = C_{\mathrm{LC}} \, N^{\mu} \, (\Delta\varepsilon)^{b} \, e^{(T-T_0)/B} \tag{3}$$

where:
d – thickness decrement [mm], T – temperature, N – number of cycles, σ – stress, σ_{gr} – stress limit below which stress corrosion is not observed, $\Delta\varepsilon$ – strain range, C_{SC}, n, B, C_{HC}, κ, m, C_{LC}, μ, b – model constants that are to be calibrated in one-dimensional experiments (JAKUBOWSKI 2000).

A single load cycle was divided into computational steps with very small load increments ($\Delta\lambda \leq 0.05$). After every load increment, thickness decrement was calculated as the following sum: $d^{j} = d^{j}_{\mathrm{SC}} + d^{j}_{\mathrm{HC}} + d^{j}_{\mathrm{LC}}$ (where j is the number of a truss member), and the cross-sectional area of the j-th member was updated. The updated values of the cross-sectional areas of all truss members provided a basis for updating the stiffness matrix used for calculations at successive increment steps. If plastic strain occurred in a given structural component during numerical simulation, the unit dissipation energy corresponding to this component was calculated as the work of stress on its plastic strains. This energy was compared with critical energy whose value was computed based on the empirical dependence proposed by (JAKUBOWSKI 2000):

$$U_{kr} = 0.0025\ (3R_m + R_e)\ A_{10} \tag{4}$$

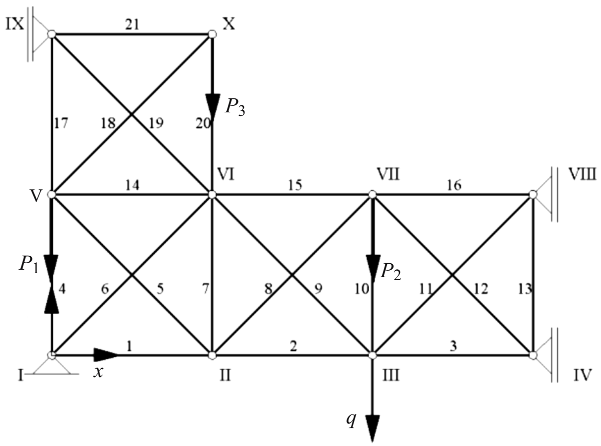
where:
R_m – tensile strength [Mpa], R_e – yield point [Mpa], A_{10} – percentage unit elongation after the tensile failure of a tenfold specimen.

If the value of dissipated energy exceeds the level of critical energy, the program stops calculations, enabling the user to introduce changes (e.g. restore the initial values of cross-sectional areas and properties of the material – regeneration of a truss member) and to continue analysis on the effect of these changes on truss behavior.

Additionally, the subprograms developed for the visualization of computation results (DUDDA 2005) permit to track the equilibrium path, structure deformation, degradation of the cross-sectional areas and energy dissipation of particular truss members.

Numerical analysis of a truss

A truss construction consisting of 21 members was analyzed in the study (Fig. 1). The initial cross-sectional areas of all members were adopted at 5 cm^2. It was assumed that all components were made of the same material, i.e. constructional steel St2.



no. of joint	coordinates	
	x [m]	y [m]
I	0.0	0.0
II	1.0	0.0
III	2.0	0.0
IV	3.0	0.0
V	0.0	1.0
VI	1.0	1.0
VII	2.0	1.0
VIII	3.0	1.0
IX	0.0	2.0
X	1.0	2.0

Fig. 1. Structure geometry

Material constants were as follows: Young's modulus $E = 2.1 \cdot 10^5$ MPa, modulus of strain hardening $E_T = 1 \cdot 10^4$ MPa, yield point $R_e = 200$ MPa, tensile strength $R_m = 420$ Mpa ultimate elongation $A10 = 29\%$.

It follows from the above values that critical energy determined by dependence (4) was $U_{kr} = 106$ Mpa.

The truss was exposed to cyclic loading: $P_1 = \lambda \cdot 40$ kN, $P_2 = \lambda \cdot 120$ kN and $P_3 = \lambda \cdot 70$ kN (Fig. 1). Load control parameter λ was modified with the increment of $\Delta\lambda = 0.05$, as shown in Figure 2. Each cycle corresponded to 100 working hours.

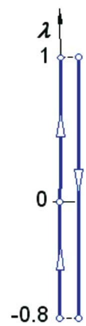


Fig. 2. Sequence of changes in load parameter λ

The parameters of corrosion models were as follows: $C_{SC} = 2 \cdot 10^{-5}$, $C_{HC} = 4.6 \cdot 10^{-6}$, $C_{LC} = 3.5 \cdot 10^{-6}$, $n = 1$, $\sigma_{gr} = 150$ MPa, $\kappa = 1$, $\vartheta = 1$, $\mu = 1.6$ and $b = 1$. Each cycle lasted for 360 hours, and it increased proportionally to $\Delta\lambda$, whereas temperature remained constant during the cycles, at 20°C. Based on the values of the above constants, the rates of corrosion of particular types were as follows: $\dot{d}_{SC} = 10^{-3}$ mm/h, $\dot{d}_{HC} = 0.4$ mm/year and $\dot{d}_{LC} = 1.5 \cdot 10^{-4}$ mm/cycle (DUDDA 2005).

Ten series of numerical computations were performed for the discussed truss. In successive series, truss member no. 4 was regenerated at a later stage. The number of load cycles after which truss member no. 4 was regenerated is shown in Table 1.

Table 1

Computational series and the number of cycles during which truss member no. 4 was replaced

Series number	I	II	III	IV	V	VI	VII	VIII	IX	X
Number of cycles N_w until truss member no. 4 was replaced	60	70	80	90	100	110	120	130	140	150

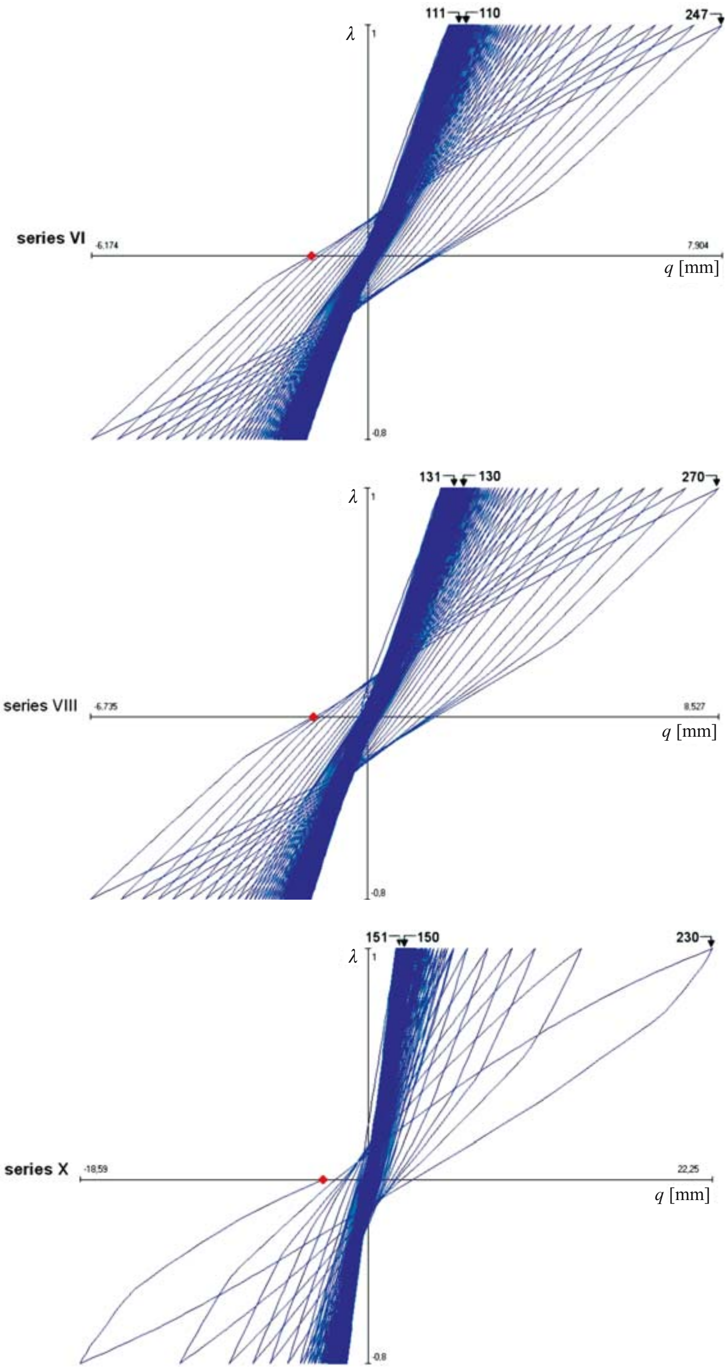


Fig. 3. Equilibrium path of control displacement $q = f(\lambda)$

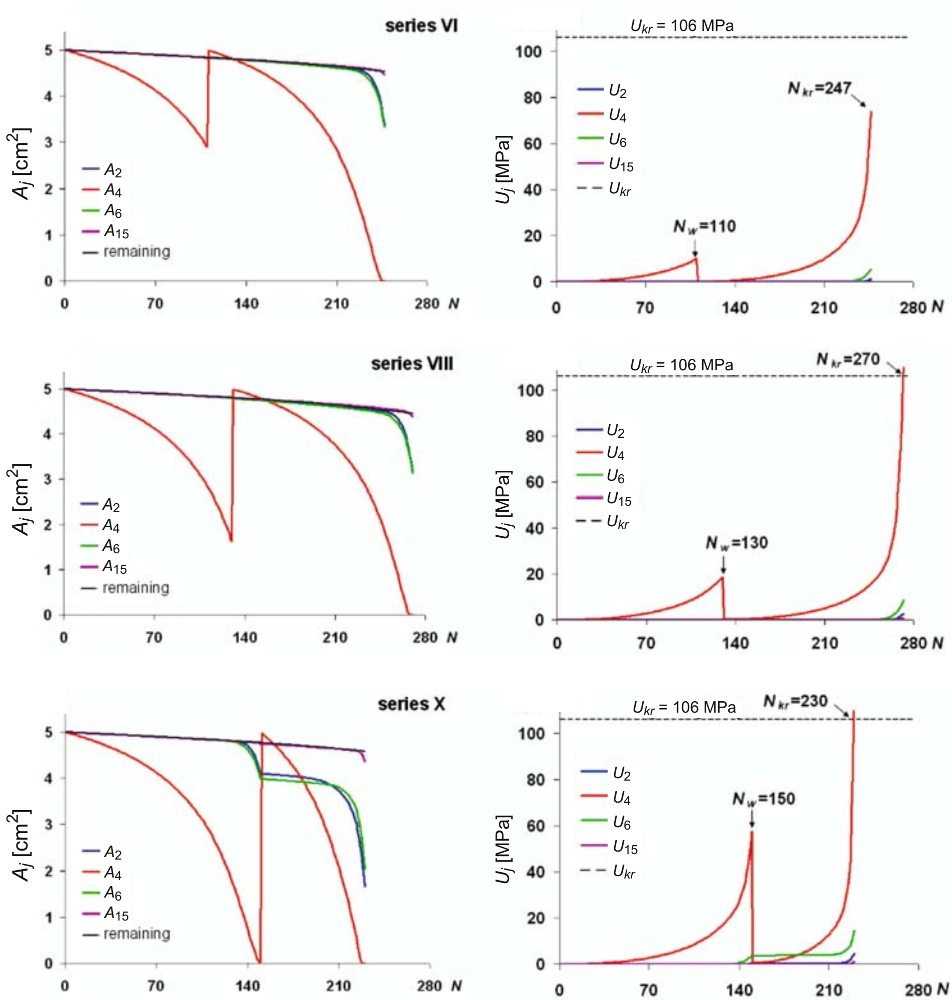


Fig. 4. Degradation of the cross-sectional areas of truss members A_j and energy dissipation U_j as dependent on the number of cycles N

In each series, after the replacement of truss member no. 4 for a new one, the computations were continued until energy dissipated by any member exceeded the critical level or until the cross-sectional area of any member underwent total corrosive degradation.

The relationship between control displacement q (Fig. 1.) and load parameter λ, as well as the increments of dissipated energy E_i and changes in the cross-sectional areas of truss members A_i, were monitored during numerical simulation. The results of computations for series VI, VIII and X are presented in Figures 3 and 4.

Results and discussion

In order to determine the effect of the moment of component regeneration on the behavior of truss construction during successive load cycles, the results of all computational series were presented in two figures, 5 and 6. Figure 5 shows the distribution of the critical number of load cycles until the moment of total degradation as dependent on the number of cycles after which a given component was regenerated. Figure 6 shows the distribution of the highest values of control displacement q for cycles immediately before and after the replacement of the degraded component, and for the critical number of cycles.

An analysis of Figure 5 indicates that when truss member no. 4 was regenerated after 130 cycles, the construction could be subjected to further loading until cycle 270. The replacement of this component during the remaining series resulted in lower values of the critical number of cycles.

The distribution of the maximal values of control displacement (Fig. 6) shows that if truss member 4 was replaced between cycle 60 and 140, the maximal control displacements corresponding to the critical number of cycles remained within the range of 6 to 9 mm. The regeneration of truss member no. 4 after 150 cycles led to a rapid increment in control displacement, which reached the value of 22.25 mm. Figure 6 also shows that the moment of component regeneration had no effect on the values of control displacement in the cycles following component replacement.

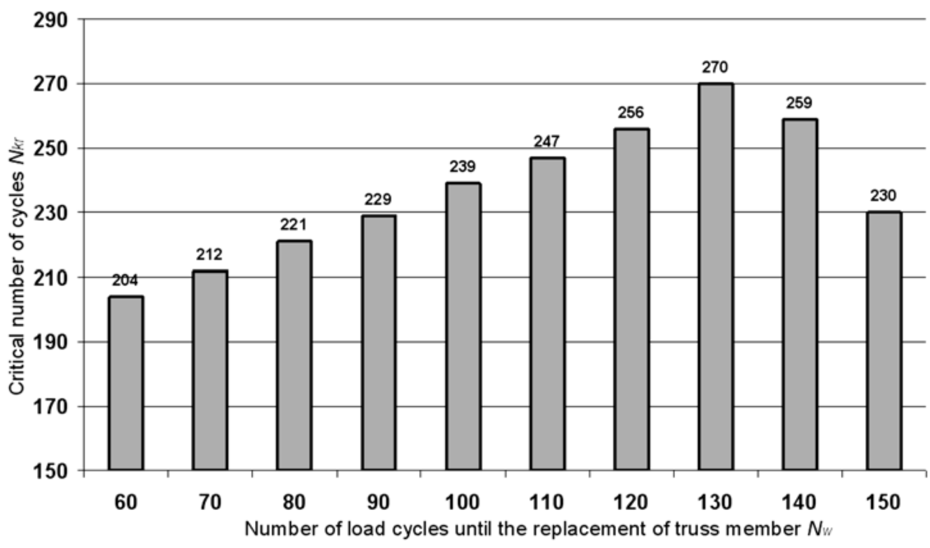


Fig. 5. Critical number of cycles N_{kr} in successive computational series

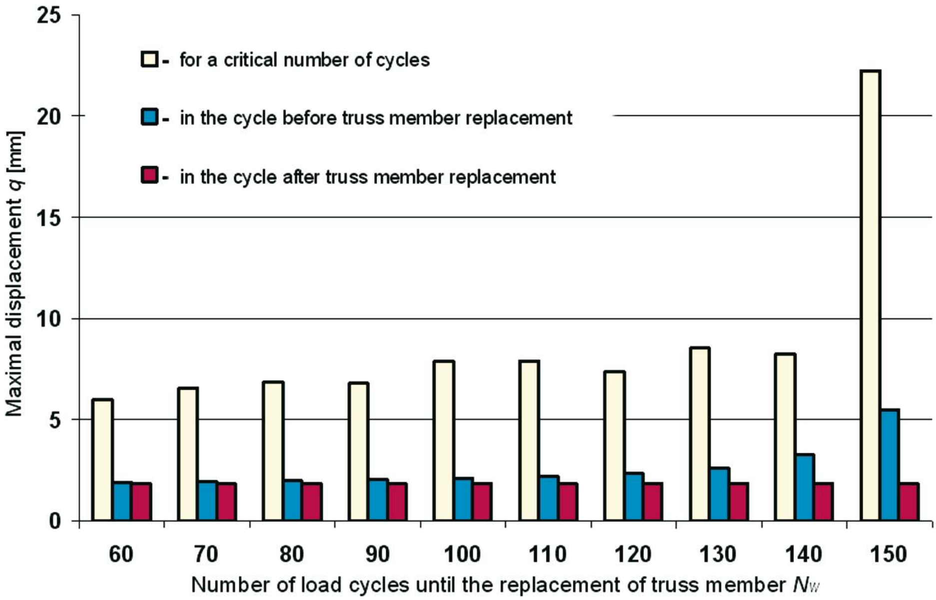


Fig. 6. Distribution of the maximal values of control displacement q

As illustrated in Figure 6, the replacement of truss member no. 4 until cycle 130 was not followed by significant changes in the value of maximal displacement. Considerable changes occurred if said component was replaced after cycle 130. Therefore, the optimal moment for truss member replacement was cycle 130, as confirmed by the energy-based criterion which was adopted as the main criterion for estimating construction durability. Another appropriate moment for the replacement could be cycle 140. However, it should be noted that in subsequent cycles the truss construction comes closer to the moment of stability loss and damage. A rapid increment in displacement observed when said component was replaced after 140 cycles could increase the risk of stability loss during the process. The stability of the analyzed truss gradually decreased starting from that cycle.

Conclusions

The calculations carried out in this study and an analysis of the obtained results indicate that the tools applied in the area of structural mechanics can be used to develop numerical models that enable to simulate the process of structure degradation under cyclic loading. The results of computations provide information about the behavior of a given structure as a whole and about the degradation of particular components, which in turn allows to determine the durability of truss constructions and express it as the number of load cycles until the replacement of degraded components.

References

CHRÓŚCIELEWSKI J. 1996. *Rodzina elementów skończonych klasy C^o w nieliniowej sześcioparametrowej teorii powłok*. Zeszyty naukowe Politechniki Gdańskiej, 540, praca hab., Gdańsk.

DUDDA W., BADUR J., CHRÓŚCIELEWSKI J. 2000. *Weryfikacja metody obliczeń konstrukcji kratowej obciążonej w zakresie sprężysto-plastycznym*. Archiwum IMPPAN, 151, Gdańsk.

DUDDA W., BADUR J. 2001. *Ocena żywotności konstrukcji przy użyciu numerycznych metod mechaniki*. Wydawnictwo UWM, Olsztyn, ss. 109-119.

DUDDA W. 2005. *Numerical modelling of a structure corrosive degradation during work cycles*. Letters of Institute of Fluid-Flow Machinery, Polish Academy of Sciences, 1456: 1-144, Gdansk.

JAKUBOWSKI M. 2000. *Mechanizm propagacji pęknięć korozyjno-zmęczeniowych w stali austenitycznej zbliżonej do AISI 316LN*. Materiały XVIII Sympozjum Zmęczenia Materiałów i Konstrukcji, Bydgoszcz-Pieczyska, pp. 179-186.

KOCAŃDA S., SZALA J. 1997. *Podstawy obliczeń zmęczeniowych*. WNT, Warszawa.

Accepted for print 16.10.2008 r.