Assessment of foF2 and hmF2 monthly median variability using COSMIC radio-occultation profiles

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Motivation

This work aims to be a contribution toward developing an empirical model of the standard deviation of instantaneous values from the monthly mean of the NmF2 and hmF2 parameters computed by the IRI model.
**Error estimation procedure**

The error estimation procedure relies upon comparing the modeled values to the ones retrieved from FormoSat-3/COSMIC radio-occultation profiles (ROP):

\[ \Delta = \Omega_M - \Omega_R \]

where

- \( \Omega \) = either \( N_m F_2 \) or \( h_m F_2 \) parameter;
- \( \Omega_M \) = modeled value computed with IRI, i.e.: monthly median value of \( f_0 F_2 \) or \( M_{3000} F_2 \) computed according to the ITUR recommendation, and converted to \( N_m F_2 \) or \( h_m F_2 \) with the Bilitza formula (ref. 1–7);
- \( \Omega_R \) = retrieved value, i.e.: instantaneous values of \( N_m F_2 \) or \( h_m F_2 \) retrieved from the ROP using the procedure that will be described in the next viewgraph.
**Procedure for retrieving NmF2 and hmF2 from ROP**

ROP downloaded from CDAAC database at UCAR (http://cdaac-www.cosmic.ucar.edu/cdaac/products.html) are individually fitted with the La Plata Ionospheric Model (LPIM) (ref. 7).

LPIM uses 3 $\alpha$-Chapman layers to represent the electron density as function of the height in the E, F1 and F2 layer, and a vary-Chapman function in top-side.

LPIM is parameterized as a function of the electron density, height, and scale height of the F2 layer.

A re-weighted Least Squares algorithm (based on the ‘bisquare’ weighting function) is used for down-weighting unreliable data (occasionally, entire ROP) and for retrieving the model parameters and their variances.

We tuned the algorithm until it did not accept data that must be indisputably discarded and did not discard measurements that must be indubitably accepted.

These procedure assigned negligible weights (lower than one tenth of the unity of weight) to approximately 20% of the data (including complete ROP).

About half of cases coincided with data (and complete ROP) that are indisputably unrealistic; in the other cases, it is difficult to ascertain whether the misalignment between data and LPIM should be attributed to problems in the data, LPIM or both.
**Working hypothesis**

The difference between the IRI and retrieved value can be split into two components:

\[ \Delta = \varepsilon_M + \varepsilon_R \]

- Error of the retrieved value
- Error of the modeled values

\[ \varepsilon_M = \varepsilon_B + \varepsilon_D \]

- Monthly mean constant bias
- Day-to-day deviation w.r.t. the monthly mean bias

Putting all together:

\[ \Delta = \varepsilon_B + \varepsilon_D + \varepsilon_R \]

- Mean \( \Delta \) = \( \varepsilon_B \)
- Variance \( \Delta \) = \( \text{var}(\varepsilon_D) + \text{var}(\varepsilon_R) \)

\[ \text{var}(\varepsilon_D) \]

- Can be modeled as a randomly distributed variable with zero mean and variance

\[ \text{var}(\varepsilon_R) \]

- Can be modeled as a randomly distributed variable with zero mean and variance
Retrieve $\Omega_R$ and $\text{var}(\varepsilon_R)$ instantaneous values from ROP

Compute $\Omega_M$ monthly median value with IRI

Compute the $\Delta = \Omega_M - \Omega_R$ differences

Estimate the monthly mean model bias $\varepsilon_B = \text{mean}(\Delta)$

Estimate the variance of the differences $\text{var}(\Delta)$

Estimate the variance of the day-to-day deviation $\text{var}(\varepsilon_D) = \text{var}(\Delta) - \text{var}(\varepsilon_R)$
**Numerical procedure**

The monthly mean bias and the variance of the day-to-day deviation are described in terms of 5 coordinates:

\[ \varepsilon_B = f(m, IG, A_p, LT, \mu) \]
\[ \text{var}(\varepsilon_D) = g(m, IG, A_p, LT, \mu) \]

The computed differences are binned into a 5-D grid and the statistical parameters are computed within each beam:

\[ \varepsilon_B = \frac{1}{n} \sum \Delta \]
\[ \text{var}(\varepsilon_D) = \frac{1}{n-1} \sum (\Delta - \varepsilon_B)^2 - \text{var}(\varepsilon_R) \]

The following girding criteria has been applied in this research:

<table>
<thead>
<tr>
<th>Param</th>
<th>From</th>
<th>To</th>
<th>Step</th>
<th>Param</th>
<th>From</th>
<th>To</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>1</td>
<td>12</td>
<td>1</td>
<td>LT</td>
<td>-1h</td>
<td>23h</td>
<td>2h</td>
</tr>
<tr>
<td>IG_{12}</td>
<td>-20</td>
<td>20</td>
<td>40</td>
<td>\mu</td>
<td>-60º</td>
<td>60º</td>
<td>10º</td>
</tr>
<tr>
<td>A_p</td>
<td>0</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Dataset**

Results for NmF2 at LT 11

Low Solar Activity (-20 < IG_12 < 20)

Quiet Geomagnetic Conditions (Ap < 15)

Monthly mean model bias (% of the IRI value)

Variance of the day-to-day deviation (% of IRI value)

NmF2 computed with IRI ($10^{10}$ elect/cm$^3$)
Results for NmF2 at LT 23

- 01

Low Solar Activity (-20 < IG_12 < 20)

Quiet Geomagnetic Conditions (Ap < 15)

Monthly mean model bias (% of the IRI value)

Variance of the day-to-day deviation (% of IRI value)
Results for hmF2 at LT 11 - 13

Low Solar Activity (-20 < IG_12 < 20)

Quiet Geomagnetic Conditions (Ap < 15)

Monthly mean model bias (% of the IRI value)

Variance of the day-to-day deviation (% of IRI value)
Results for hmF2 at LT 23 - 01

Low Solar Activity (-20 < IG_12 < 20)

Quiet Geomagnetic Conditions (Ap < 15)

Monthly mean model bias (% of the IRI value)

Variance of the day-to-day deviation (% of IRI value)
Summary and conclusion

An attempt was made to establish an empirical model to predict the error of the NmF2 and hmF2 parameters computed by IRI using FormoSat-3/COSMIC radio-occultation profiles.

The differences between the modeled and retrieved parameters was explained in terms of three contributions:

• errors in the retrieved parameters;

• a constant bias in IRI predictions; and

• random errors in IRI predictions due to the unaccounted day-to-day variability.

The errors of the retrieved parameters were estimated using the LPIM model;

The bias and the day-to-day variability of IRI were represented as functions of the month, solar activity, geomagnetic perturbation, modip and local time.
**Summary and conclusion**

Quiet geomagnetic days in a low solar activity period was analyzed; the obtained results are summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>NmF2</th>
<th>hmf2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>bias</strong></td>
<td>day-to-day variation</td>
<td>bias</td>
</tr>
<tr>
<td><strong>noon time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35%, IRI</td>
<td>±10% to ±30%; maximums on the</td>
<td>-4% to +12%, IRI overestimation</td>
</tr>
<tr>
<td>overestimation</td>
<td>northern and southern sides of</td>
<td>over the equator and mid</td>
</tr>
<tr>
<td>maximums on</td>
<td>the EA</td>
<td>latitude; underestimation over</td>
</tr>
<tr>
<td>the crests of</td>
<td></td>
<td>the crest of the EA.</td>
</tr>
<tr>
<td>the Equatorial Anomaly (EA)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>mid night</strong></td>
<td>±20% to ±45%</td>
<td>-8% to +2%, IRI</td>
</tr>
<tr>
<td>50%, IRI</td>
<td></td>
<td>underestimation in general but</td>
</tr>
<tr>
<td>overestimation</td>
<td></td>
<td>overestimation at mid</td>
</tr>
<tr>
<td></td>
<td></td>
<td>latitude</td>
</tr>
</tbody>
</table>

The method employed in this research seems to be appropriated for the intended purpose. Much more work will be needed to establish its validity and reliability.
References

The La Plata Ionospheric Model (LPIM)

The LPIM version used in this research follow the inspiration of the IRI and NeQuick models in the sense that it uses:

- an empirical representation of the geographical (global) and temporal variability of the F2 layer parameters ($N_{m,F2}$, $h_{m,F2}$ and HF2);
- a semi-empirical representation of the vertical profile of the ED anchored to the above mentioned F2 layer parameters.
Global models for the variation of the F2-peak parameters with latitude, longitude and time

The F2 layer parameters: $N_mF_2$ (electron density of the F2 peak); $h_mF_2$ (height of the F2 peak); and $HF_2$ (scale height of the F2 layer), are separately modeled with spherical harmonics expansions dependent on the modip latitude and local time, with time (UT) dependent coefficients:

$$\Omega(\mu, \lambda, t) = a_0(t) + \sum_{l=1}^{L} \sum_{m=1}^{l} \left[ a_{lm}(t) \cdot \cos\left( m \cdot \frac{2\pi}{24} \cdot \lambda \right) + b_{lm}(t) \cdot \cos\left( m \cdot \frac{2\pi}{24} \cdot \lambda \right) \right] \cdot P_{lm}(\sin \mu)$$
Model for the variation of the electron density with height

The vertical profile is modeled as a superposition of three Chapman layers for the E, F1 and the bottom-side of the F2 layer and a vary-Chapman layer for the topside.

The main parameters of the E, F1 and for the topside layers are anchored to the parameters of the F2-layer, and modeled in accordance to the ITU-R recommendations.

\[ N_e(h) = \sum_{i=1}^{3} N_{m,i} \cdot e^{\frac{1}{2} \left[ 1-z_i - e^{-z_i} \right]} \]

\[ N_e(h) = N_m F2 \cdot \sqrt{\frac{H(h_{m,F2})}{H(h)}} \cdot e^{\frac{1}{2} \left[ 1-z(h) - e^{-z(h)} \right]} \]

\[ z(h) = \int_{h_{m,F2}}^{h} \frac{d\zeta}{H(\zeta)} \]
The LPIM profile is fitted, by Least Squares, to every measured ROP.
The fitting is done by adjusting the three parameters of the F2 layer: NmF2, hmF2 and HF2.
The Least Squares method allows computing estimate of the parameters along with estimates of their errors (by variance – covariance propagation of the observed minus modeled deviations).

~ 60,000 profiles like this one per month

σ = ±4.5*10^{10} \text{ m}^{-3}
**Example of computed minus modeled electron densities**

Standard deviation of the computed minus modeled electron densities for the ROP comprised within the 18 – 20 UT interval

Sep 2007 Values in $10^{10}$ m$^{-3}$ Dec 2011

Standard deviations range around 3% of the measured ED
Example of the estimated NmF2 parameters and standard deviations

Standard deviations (obtained by variance – covariance propagation) range around 7% of the estimated value
Example of the estimated $h_m F_2$ parameters and standard deviations

Standard deviations (obtained by variance – covariance propagation) range around 1% of the estimated value